## Precollege Mathematics and Engineering Learning Outcomes: Implications for Equitable Preparation, Recruitment, and Retention\*

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The importance of precollege mathematics achievement is well documented, but the complex relationships between different measures of precollege mathematics achievement and engineering outcomes has not been fully explored. The purpose for this study was to better understand the interrelationship between common measures of precollege mathematics achievement and success in post-secondary engineering. The principal research question was "How are precollege mathematics achievement measures associated with intercorrelated engineering course success for those who graduated with an engineering degree?" We used extant enrollment and transcript data in our canonical correlation analysis to assess the relationship between two variant sets: measures of precollege mathematics achievement and engineering course success. The precollege mathematics variant set was statistically significantly related to the engineering learning outcomes. Two canonical functions were retained and examined: (1) Mathematics Placement Calculus BC scores and time to graduation were the most influential factors. The results contribute a parsimonious model of mathematics achievement variables useful in capturing prerequisites for successful completion of an engineering degree. We provide evidence that there is no meaningful relationship between taking and passing the Calculus AB or BC exams and time to graduation and credit earned. These results can be useful to school counselors and science, technology, engineering, and mathematics teachers for advising high school students.

Keywords: canonical correlation; mathematics achievement; engineering education preparation; retention; calculus; ACT; SAT

## 1. Introduction

Considerable attention has been devoted to examining factors that influence matriculation and retention of engineering students. One result of this interest is that there exists consistent and credible evidence that precollege mathematics success is requisite for both matriculation and retention [1-4]. Students who are underprepared in mathematics either do not choose to matriculate into postsecondary programs or experience a much higher than normal dropout rate due to requirements to take non-credit bearing courses and increased time to graduation [5], as well as delays in taking courses in their subject area of interest. The nature of precollege mathematics success and what this notion entails must be deeply discussed however, as the available support systems and experiences that influence academic success are different for every student. In fact, a renewed interest in studying students' opportunities to learn mathematics has come about in response to the need to diversify the engineering workforce.

The types and amount of admission data colleges receive from students often reflect differences in opportunities to learn mathematics. Opportunities to learn can be characterized as "what students learn in school is related to what is taught in school" [6, p. 541]. These opportunities to learn are often equated to the quality of resources and instruction a student receives or has access to in school settings, which varies drastically. Students with more opportunities to learn often have multiple measures of precollege mathematics achievement compared to students with fewer opportunities.

Historically, precollege mathematics success was measured by student performance on the mathematics sections of the Scholastic Aptitude Test (SAT) and the American College Test (ACT). However, because calculus is an essential engineering mathematics content strand [7], many engineering programs within selective colleges and universities often consider Advanced Placement (AP) Calculus scores as necessary predictors of student retention and success in engineering. The AP Calculus exam is administered in two versions, AB which covers the content of one semester of calculus while the BC exam covers the content covered in both semesters of calculus. This emerging emphasis is problematic given that not all students have access to AP Calculus and, more

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importantly, traditionally underrepresented students of color often disproportionately lack access to these advanced mathematics opportunities [8], thus placing them at a disadvantage during the admissions process.

The role of college admission exams on engineering retention has been extensively examined over the last two decades [9-12]. What remains underexamined is the interrelationship between different measures of precollege mathematics achievement and pertinent engineering learning outcomes. Differences in opportunities to learn mathematics in high school cannot be controlled despite differential effects on admission into and success in colleges of engineering. However, colleges of engineering are well equipped to make more informed admission decisions based on scores from precollege mathematics assessments [9, 12]. Thus, the purpose for this study was to examine the interrelationships between a set of precollege measures of mathematics achievement and a set of engineering learning outcomes. These relationships have important implications for the preparation, recruitment, and retention of a more diverse engineering workforce [13, 14]. Therefore, we examined how precollege mathematics achievement measures are associated with intercorrelated engineering learning outcomes for those who graduated with an engineering degree.

# 2. Precollege Mathematics Achievement and Engineering Success

Early access to advanced mathematics is considered one of the most important opportunities to learn within U.S. schools. It is commonly understood that students who have access to high-quality mathematics preparation courses in high school are often better prepared for post-secondary engineering mathematics [13, 14]. This is not the only predictor of engineering success, but few researchers have attempted to understand the nexus between multiple measures of precollege mathematics achievement and engineering success [15, 16]. Calculus readiness is arguably one of the most important considerations for admission and retention for colleges of engineering [13, 17]. This relationship is analogous to the relationship between success in eighth-grade algebra and success in high school mathematics. Historically, researchers observed that college admission exams were good predictors of calculus readiness and subsequent engineering success [17]. Many scholars have argued that the direct measurement of calculus readiness is a better predictor of student preparation for engineering [13, 14]. The direct measurement of calculus readiness is often assessed by

standardized tests and college-designed calculus readiness examinations. A current trend is that universities are developing their own measures of calculus readiness.

### 2.1 College Admissions Exams

A combination of SAT and ACT scores and high school grades has consistently been shown to be the best predictor of freshman college engineering success [18, 19]. These measures have not been universally implemented; across studies researchers frequently use different sets including ACT/SAT scores, rank in high school, SAT Math Test scores, ACT scores, and GPA, as predictors [17-21]. In a meta-analysis, there was a statistically significant long-term retention effect of 0.36 for SAT mathematics scores [22]. Overall, using standardized admission tests (e.g., SAT and ACT) and GPA as measures of academic preparedness has been shown, with limited success, to be associated with engineering success and useful for admissions decisions.

#### 2.2 Credit Equivalency Exams

Early access and achievement in engineeringrelated mathematics, specifically calculus, is another important precollege measure of mathematics achievement that warrants consideration. AP Calculus courses provide students a means to enter colleges and universities with earned credits in engineering-related mathematics. In an examination of 10 years of admission and completion data, Bowen et al. [13] found that calculus readiness was a statistically significant predictor of engineering graduation rates. Participation in either AP Calculus AB or BC was the strongest predictor of subsequent achievement in college engineering-related science and mathematics [23].

Historically, AP course access was restricted to a select group of "superior" high school students. Often, these students were judged by their teachers to be capable of engaging with work aligned to university curricula. Because AP Calculus has been shown to be a statistically significant predictor of post-secondary engineering courses, students who are denied access to AP Calculus may also be denied an opportunity to enter engineering careers. Admission to AP courses is typically determined by a student's prior achievement and teacher recommendations. Even today, AP Calculus is often restricted to the top 5 to 10% of students. Therefore, other precollege measures of mathematics achievement may better reflect the abilities of a more diverse population of potential engineering students. Some argue that the academic superiority of the students taking AP exams mediates the relationship between student AP Calculus achievement and engineering success [24]. To circumvent this challenge, many exclusive colleges require potential STEM majors to complete a calculus readiness exam prior to placement in or exemption from college calculus.

#### 2.3 Calculus Readiness Exams

Entry into the engineering calculus sequence by a calculus readiness exam is an emergent trend. This is partly informed by the observed moderate to large effects within models examining the relationship between calculus readiness measured by precollege examinations and engineering retention [25]. The relationship between success in an initial calculus course and student retention in engineering has only increased national attention and support for this practice [26]. Mandatory calculus readiness exams partly function by enabling advisors to guide freshman STEM majors to enroll into appropriate mathematics course sequence. Understanding the relationship between different measures of calculus readiness and student success is important because this knowledge could be leveraged to create alternative models for admitting talented engineering students.

## 3. Problem Statement

Recruiting and retaining talented engineering students is an arduous and complex task. One of the most challenging aspects of this task is identifying engineering talent effectively, efficiently, and equitably. Participation trends indicate that U.S. high school students take substantially more college admissions exams (e.g., SAT and ACT) compared to AP exams. Compared to the approximately 1.9 million students who completed the SAT in 2023 [27], a little more than 400 thousand completed either the Calculus AB or BC exam in 2023 [28]. Thus, students complete about four times as many SAT exams compared to AP Calculus exams. It would be remiss not to acknowledge that the SAT is considered one of the standard exams for college admission, while AP exams tend to measure mastery of college-level material. Nonetheless, the participation trends for SAT exams demonstrate the potential for engineering colleges to select from a larger and possibly more diverse pool of learners should these test scores be shown to contribute effectively either directly or indirectly as a mediator to overall model prediction.

Moreover, traditionally underrepresented populations of potential engineering learners, such as students of color, women, and first-generation students, are more likely to have an SAT or ACT mathematics score than an AP Calculus score given that the SAT/ACT exams are more affordable and have fewer barriers to access [29]. Thus, understanding the nature of the relationship between these and other precollege mathematics achievement measures can potentially inform efforts to recruit and retain a more diverse population of engineering students.

The desire to predict a student's potential to succeed in engineering-related mathematics is one of the most important factors in admission decisions. The majority of extant data have focused on measures of mathematics achievement in isolation rather than examining the combined and unique relationships between different measures of precollege mathematics achievement and measures of engineering success. Understanding the relationship between a student's precollege mathematics achievement profile and pertinent measures of engineering success is critical to selecting students on their propensity for success regardless of their access to specific opportunities to learn mathematics. Here, we focus on college admissions exams (i.e., ACT and SAT), AP mathematics exams, and calculus readiness tests as three measures that constitute a student's mathematics achievement profile. Only in direct comparisons is it possible to disentangle both the unique and combined effects of these key predictors.

Our aim is to better understand the combined relationships between these academic achievement measures and three measures of engineering success pertinent to students, administrators, faculty, and parents. These three engineering achievement measures are as follows: GPA, time to graduation, and credits earned. We selected these factors because they have practical implications that resonate with administrators, faculty members, students, and parents. Essentially, all four constituents would like for students to graduate faster, learn more by completing more credits, and demonstrate comprehension as measured by GPA. By examining the interrelationships between these two sets of factors, we can inform college preparation advisors and those who recruit potential students to obtain a more representative population of potential engineers. Thus, the overarching research question that framed this study is the following: How are precollege mathematics achievement measures associated with intercorrelated engineering learning outcomes?

## 4. Method

#### 4.1 Participants

Extant transcript and enrollment data for undergraduate engineering students (n = 2,322) from a university in the Southwestern United States were analyzed for the present study. The majority of the participants were male (74.2%) and non-first-generation college students (78.8%). Although the majority of the participants were White (59.7%), there was representation from Asian (11.8%), Latinx (22.2%), and Black (2.0%) students. The remaining students were racially unidentified (4.3%). All of the participants were true freshman, non-transfer students, and data were collected from their first semester until graduation from the College of Engineering.

#### 4.2 Precollege Achievement Measures

Five measures of mathematics achievement were utilized to assess the relationship between mathematics achievement and engineering learning outcomes. The first two measures were the mathematics sections of two college entrance examinations (i.e., SAT and ACT). The SAT is a multiple-choice paper-and-pencil exam designed to measure a high school student's college readiness. The SAT comprises two main sections – the mathematics section and Evidence-Based Reading and Writing. There is an optional essay section that can be included in a student's assessment for an additional fee. The mathematics section is scored from 200 to 800 [30].

The ACT is also a multiple-choice paper-andpencil exam; however, unlike the SAT, the ACT has four sections (i.e., mathematics, reading, science, and English). Each section is scored on a 36-point scale [31]. The third and fourth measures utilized in the present study were the AP Calculus AB and BC exams.

The AP courses are college equivalent classes that students can take in high school to earn college credit by successfully passing the AP exam. According to the College Board, Calculus AB represents a single semester of calculus and Calculus BC is equivalent to two semesters or a year of calculus [28]. Both exams contain multiple-choice and freeresponse items and are scored on a scale from 1 (*No Recommendation*) to 5 (*Extremely Qualified*) [32]. Because the Calculus BC exam represents two semesters of calculus, students receive a Calculus AB subscore that represents the students' performance on the 60% of the material on the Calculus BC exam that measures content from the Calculus AB exam.

The fifth measure of precollege mathematics achievement was a mathematics placement exam designed by the Department of Mathematics at the university. Officially titled the Math Placement Exam (MPE), the exam takes approximately 90 minutes to complete and is used to assess students' requisite mathematics skills to pursue calculus. As such, the MPE was developed as a calculus readiness exam. This is an important tool to help advisors determine which mathematics courses incoming students should take. In fact, the College of Engineering and the College of Science (with the exception of students from the Department of Biology) require all incoming freshmen to take this exam regardless of prior indicators of mathematics success. Incoming students take the MPE online and are proctored by university personnel. Practice problems can be accessed as freely available PDF files.

#### 4.3 Engineering Learning Outcome Measures

Success in engineering was assessed using three learning outcomes derived from student transcript data. According to the American Society of Engineering Education [33], only 33% of engineering students complete their degrees in four years. Thus, the first measure was time to graduation measured in years, from the first semester of enrollment to graduation. The second measure was college GPA as reported on the student's transcript. Because all participants in the present study graduated, GPA is used to represent the student's level of mastery or achievement. The third engineering learning outcome measure was credits earned. All undergraduate engineering degrees require 128 credit hours. The relationship between each of the measures and the two variates is presented in Fig. 1. The first variate (i.e., precollege mathematics achievement) consists of five mathematics achievement measures. The second variate (i.e., engineering success) consists of three engineering learning outcomes. The canonical correlation is the correlation between the two variable sets or variates, represented by the double-headed arrow connecting the two variates.

#### 4.4 Data Analysis

To examine the relationship between precollege mathematics achievement and engineering learning outcomes, a canonical correlation analysis (CCA) was performed. A CCA is a statistical technique for examining the multivariate relationship between two sets of two or more constructs/variables [34]. The goal of using a CCA is to unpack the relational patterns present between two canonical variates (i.e., two distinct variable sets combined to form a pair). According to Thompson [35], CCA is a unified approach to many univariate and multivariate statistical procedures. Moreover, CCA has been considered the second most encompassing analytic technique within the general linear model [36-37], only surpassed by structural equation modeling. A CCA can be used to replicate any other (except for structural equation modeling) analytic technique in the general linear model [38].

A CCA was chosen because our purpose was to assess relationships between a set of precollege mathematics achievement predictor variables and

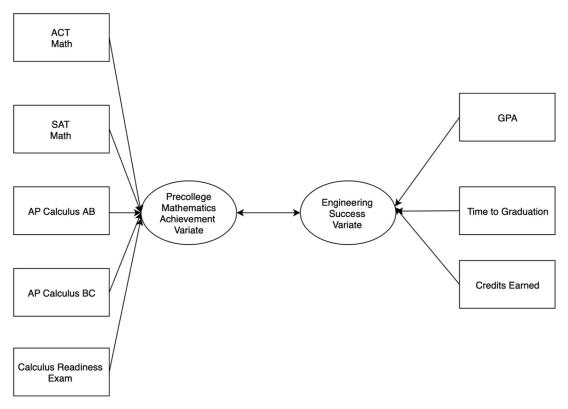


Fig. 1. Diagram of Relationship Between Measures and Variates in Canonical Correlation.

a separate set of variables representing engineering learning outcomes and to disaggregate the unique and common variance of each variable and unfortunately structural equation modeling cannot provide this information. Note that due to the correlational nature of CCA, determination of predictor and outcome variables is essentially arbitrary [39]. However, due to the theoretical relationship between mathematics preparation and engineering success, we argue that it is reasonable to assume that precollege mathematics success measures can be considered predictors of engineering learning outcomes. A CCA will, however, not facilitate the identification of causal relationships. Because our objective was to assess the relationship between variable sets, a CCA was the most appropriate analytic technique because it allowed us to identify the shared and individual contributions to the relationships presented in our model [40]. Furthermore, the multivariate nature of a CCA limits the probability of committing a Type I error [41, 42]. In theory, we could have conduct three separate regression analyses: one for examining the relationship between precollege mathematics achievement measures and GPA, then a subsequent analysis of credits earned, and a final analysis of time to graduation. This would have increased the "test wise" error rate, however; thus, doing so was avoided.

#### 4.4.1 Limitations of CCA

Canonical correlation analysis is an underused technique because it is not well understood. This is because, all too often, this multivariate technique is not taught or covered in depth for those who are not in a Research Methods graduate program [43]. Generally, in a CCA, a normal distribution of the variables is not strictly required when canonical correlation is used descriptively; however, multivariate normality can enhance the robustness for predictive models. Homoscedasticity implies that the relationship between two variables is constant over the full range of data, and this increases the accuracy of canonical correlation. In this case, this is an affordance over ordinary least squares (OLS) analyses, because this is one of the assumptions that the Gauss-Markov theorem applies, and OLS provides the best linear unbiased estimator only when heteroscedasticity is controlled [44]. Homoscedasticity in CCA is not required for the coefficient estimates to be unbiased, consistent, and asymptotically normal, but it is required for OLS to be efficient [44].

Linearity is an important assumption of canonical correlation; this technique finds linear relationships between variables within a set and between canonical variate pairs between sets because nonlinear components of these relationships are not recognized and so are not captured in the analysis. This is an artifact of the theoretical model. If prior studies have invoked less robust linear techniques (those subsumed by CCA in the general linear model) for the same variable, then the theoretical model is already tested and accepted. In rare instances, transformation may be useful to increase linearity, but this will have consequences for any underlying theory of the relationship among the variables and should only be undertaken with clear caveats.

Perhaps the most important concern for CCA is that of outliers. Outliers have a disproportionate impact on the results of the analysis, and each set of variables must be inspected independently for univariate and multivariate outliers. Because of this very important limitation, it is necessary to avoid patterns in missing data. In general, the pattern of missing data is more important than the amount. Because canonical correlation is very sensitive to small changes in the data set, the decision to eliminate cases or estimate missing data must be considered carefully and avoided when possible.

## 4.4.2 CA Procedure

After obtaining statistically and practically significant results from our CCA, we performed a Commonality Analysis (CA), which is a method of variance partitioning designed to identify proportions of variance in the dependent (criterion) variable that may be attributed uniquely to each of the independent (predictor) variables as well as proportions of variance that are attributed to various combinations of independent (predictor) variables [37]. Although negative commonalities are possible and may indicate the presence of a suppressor effect, they should be treated as zero. There are two pros to CA: (1) it is not dependent on the order of entry of variables into the analysis, and (2) it provides accurate information about the variance accounted for by each variable without variable overlap. Both unique and common contributions to explained variance are estimated without respect to some entry; therefore, variance explained is not automatically attributed to variables entered earlier.

w>The CA was completed using agreed upon procedures [45]. The synthetic canonical variate scores were computed and the standardized canonical functioncoefficients were multiplied by the Z-scores for the measured variables in the criterion variable set to estimate the scores on the unmeasured synthetic variables, named Crit1 and Crit2. We then calculated the regressionequationsthatpredictthecriterioncomposite scores for all possible combinations of the predictor variables. Those results were then used to calculate the unique and common variance components for each predictor variable on each composite. Thiswascalculatedusingaspreadsheet. The number of components in an analysis equaled (2k-1), where k equals the number of predictor variables in the set (i.e., X predictors, Y components, Z-first order unique components). Furthermore, there are additional order variables given the total number of variables. For example, the second order is common to two variables, the third order is common to three variables, and the fourth order is common to all.

## 5. Results

Descriptive statistics for each of the precollege mathematics achievement measures and the engineering learning outcomes are presented in Table 1. The data presented in Table 1 indicated that the participants in the current study achieved mean levels of mathematics achievement commensurate with above-average performance based on annual national trends on respective mathematics measures. Because all quantitative analytic techniques are correlational in nature [46], except for Fuzzy Set Social Science, our results are an expression of the correlational relationship revealed through the analysis and represent those internal relationships.

The largest correlations were observed between the ACT and SAT, followed by the MPE and the SAT. All correlations presented in Table 1 were statistically significant (p < 0.05). On the surface, it

	Μ	SD	1	2	3	4	5	6	7	8
1. ACT Math	29.42	3.32	1							
2. SAT Math	687.75	65.08	0.72**	1						
3. AP Calculus AB	3.65	1.38	0.42**	0.48**	1					
4. AP Calculus BC	3.88	1.32	0.45**	0.44	0.79**	1				
5. Mathematics Placement Exam	23.15	11.03	0.34**	0.39**	0.21**	0.22	1			
6. GPA	3.24	0.42	0.31**	0.31**	0.35**	0.40**	0.19**	1		
7. Time to Graduation	4.07	0.85	-0.09**	-0.05	-0.11**	-0.14**	-0.21**	-0.29	1	
8. Credits Earned	137.09	23.36	0.15**	0.16**	0.13**	0.15**	0.12**	0.07**	0.004	1

Table 1. Precollege Mathematics Achievement Summary Statistics and Correlation Matrix

Note: \*\* indicates that correlation is statistically significant at the 0.01 significance level

may seem that the correlations presented are lower than expected based on prior studies. However, upon closer inspection, it is clear correlations amongst exams are relatively larger than the correlations between the engineering success measures. This is an important distinction, as most prior studies reported correlations between mathematics achievement and individual engineering courses or exams [47, 48]. The mean GPA was just above 3.0, and the mean number of years to graduation was 4.21. The average number of credits earned (138.09) was well over the minimum number of credits required to graduate (128). Time to graduation was inversely related to GPA (r = -0.29). These results indicated that as our participant's GPA increased, their time to graduation decreased; likewise, as their number of credits earned increased, time to graduation also decreased. Both relationships make intuitive sense given that earning higher grades means you earn more credits and graduate earlier.

#### 5.1 Normality and Multicollinearity

Like in all multivariate statistics, in CCA it is important to assess the data for normality and multicollinearity. We assessed distributional properties for normality, including skewness and kurtosis. The univariate skewness of the items tested ranged from –2.65 to 0.23, and univariate kurtosis varied between 1.05 and 11.34. Based on West et al.'s cutoff values, most items indicated a normal distribution, with skewness values fewer than 2 and kurtosis values under 7. However, the mathematics placement examination scores were determined to be non-normal. A log transformation was used for analysis purposes and transformed back for interpretation purposes.

The collinearity diagnostics for the precollege mathematics achievement predictor variables (i.e., ACT Math, SAT Math, Calculus AB, Calculus BC, and MPE) were acceptable. Variation inflation factor scores were substantially below 10, ranging from 1.40 to 4.19, and tolerance values varied from 0.23 to 0.71. A second set of regression analyses assessed the multicollinearity among dependent variables. The results showed variation inflation factor scores between 1.01 and 1.12 and tolerance values between 0.89 and 0.99.

#### 5.2 Relationship Between Precollege Mathematics Achievement and Engineering Learning Outcomes

To address the combined and unique relationships between precollege mathematics achievement measures (i.e., ACT Math, SAT Math, Calculus AB, Calculus BC, and MPE) and engineering learning outcomes (i.e., GPA, time to graduation, and credits earned), a CCA was conducted using precollege achievement measures as predictors of engineering learning outcomes to evaluate the multivariate shared relationship between the sets of variables. According to Sherry and Henson [36], the first step in the interpretation of a CCA is to consider the variance accounted for effect sizes in the full model to determine if there are any meaningfully important results. As such, we examined the effect sizes and relevant statistics for the entire model. The analysis yielded three functions with canonical correlation coefficients (R<sub>c</sub>): 0.41, 0.19, and 0.03. The first function  $(R_{c1} = 0.41)$  explained approximately 20.1% of the variance, and the second function ( $R_{c2} = 0.19$ ) explained approximately 3.9% of the remaining variance. The final function  $(R_{c3} = 0.03)$  explained less than 1% of the variance and was not statistically significant. Based on the contextual importance of the relationships examined in the present study, we argue that these results have practical and empirical significance that warrants further examination.

Table 2 presents the standardized canonical function coefficients and the structure coefficients for Functions 1 and 2. Due to the small correlation and variance accounted for by Function 3, we focused on relationships present in Functions 1 and 2. In CCA, the number of functions produced is limited to the smallest number of variables included in the two variable sets. Although dimension reduction is often considered in CCA, it is warranted in more complex cases where a multitude of variables are used across the two sets. Structure coefficients as well as communalities  $(h^2)$  across the two functions are also presented for each variable. Structure coefficients and communalities above 0.45 and 45%, respectively, were underlined for emphasis in Table 2. The 0.45 benchmark is an established convention used in multivariate analysis [36, 38, 49]. Based on the magnitude of the Function 1 structure coefficients (i.e., structure coefficient  $(r_s)$ values of 0.45 or above), it is evident that all five measures of precollege mathematics achievement are influential contributors to the relationship between precollege mathematics achievement and engineering learning outcomes. Based on the magnitude of the structure coefficients, MPE scores were the strongest contributors, followed by SAT and then ACT mathematics scores. Only time to graduation and GPA made notable contributions to the relationships modeled in Function 1 based on the magnitude of their structure coefficients.

Aside from the magnitude of the structure coefficients, it was also important to examine the signs. Variables with the same sign are positively related. Because all the precollege mathematics achievement measures in Function 1 had the same sign (i.e., positive), they were positively related. Con-

	Function 1			Function 2	Function 2			
Variable	Coef	r <sub>s</sub>	r <sub>s2</sub> (%)	Coef	r <sub>s</sub>	$r_{s^2}$ (%)	h <sup>2</sup> (%)	
ACT Math	0.28	0.63	39.8	-0.11	-0.19	3.6	43.4	
SAT Math	0.26	0.66	44.1	-0.42	-0.31	9.7	53.8	
Mathematics Placement Exam	0.52	0.72	52.3	0.89	-0.42	17.6	69.9	
AB Calculus	0.27	0.48	22.6	-0.24	-0.33	10.8	33.4	
BC Calculus	0.31	0.48	23.2	-0.38	0.68	46.0	<u>69.2</u>	
r <sub>c</sub>			0.41			0.19		
Time to Graduation	-0.29	-0.65	42.5	-0.89	-0.61	37.7	80.2	
Credit Earned	0.29	0.42	17.6	0.35	0.40	16.2	33.8	
GPA	0.76	0.91	82.8	-0.80	-0.39	15.1	97.9	

Table 2. Canonical Solutions for Mathematics Dispositions Predicting Mathematics Achievement 1 and 2

*Note.* Structure coefficients (*rs*) greater than |0.45| are underlined. Communality coefficients (*h*2) greater than 45% are underlined. *Coef* = standardized canonical coefficients.

versely, time to graduation was negatively related to GPA and credits earned. This relationship was logically sound, because if students had a high GPA, they earned more credits and graduated in less time. Likewise, students with lower GPAs earned fewer credits and took longer to graduate. Relatedly, GPA was the larger contributor to the relationship presented in Function 1. For our data, the relationship across variable sets indicated that all five precollege mathematics achievement measures were positively related to GPA and credits earned while inversely related to time to graduation.

The Function 2 results indicated that Calculus BC was the only notable contributor to the relationship between the two variable sets. Likewise, only time to graduation was a notable contributor within the engineering learning outcome variable set for Function 2 based on the magnitude of the structure coefficients. Furthermore, there was an inverse relationship between Calculus BC achievement and time to graduation, indicating that as student scores on the Calculus BC exam increased, the time to graduation decreased. Finally, the communalities ( $h^2$ ) indicated the magnitude of individual variable contributions across all functions.

The communality coefficient represents the sum of the squared structure coefficients for each of the functions interpreted in the analysis. Because only Functions 1 and 2 were interpreted in the present study, the  $h^2$  was essentially the sum of the squared structure coefficients for Functions 1 and 2. Therefore, the contributions to the overall model of precollege mathematics in order of magnitude were MPE scores (69.9%), Calculus BC scores (69.2%), and SAT Math Test scores (53.8%). Additionally, the communality coefficient for GPA was 97.9%, while the communality coefficient for time to graduation was 80.2%. This indicated that the influence of the variable GPA was more substantial than the contribution of the variable time to graduation across both functions. The implications of these relationships have both scientific and practical significance.

The Venn diagram (Fig. 2) provides a graphical depiction of the unique and shared contributions of MPE, Calculus BC, and SAT Math Test scores. The MPE scores from the CCA also contributed the most unique variance, followed by SAT Math Test scores. Combined these two contributed 3.7% of the variance accounted for in the model (four times the contribution of Calculus BC scores). This model clearly indicates good performance in high school calculus is less powerful in determining engineering success than scoring well in the commonly available SAT. The variance common to all variables was relatively small, in fact just smaller than that of BC Calculus scores alone. This provides additional

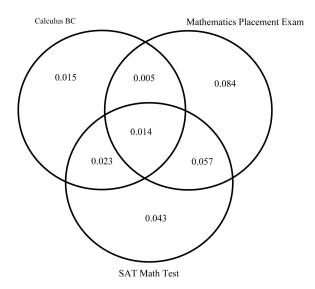


Fig. 2. Venn Diagram of the Unique and Shared Contribution of Canonical Weights.

evidence that measuring or counting BC Calculus scores may lead to disappointment to those who subscribe a large value-added component to BC Calculus enrollment. These findings raise questions about the value-added benefit to high school students from administrators stretching budgets to offer calculus. These results possibly offer a rationale for eliminating high school calculus and replacing it with a more advanced algebra class thereby reducing perceived deficits for students attending schools where there is no calculus offered.

## 6. Discussion

The goal for this study was to parsimoniously describe the number and nature of mutually independent relationships present between precollege measures of mathematics achievement and engineering learning outcomes. The results of this study suggest that two meaningful relationships exist, as evidenced by the two statistically significant canonical functions presented. In the sections that follow, we explain the nature of these relationships as they relate to the preparation, recruitment, and retention of engineering students.

#### 6.1 Canonical Function 1 Interpretation

The first canonical function represents a pathway to engineering success that is not contingent upon the completing calculus prior to college enrollment based on the contributions of each of the precollege mathematics measures to the respective relationships within and between measures of engineering success. The relationship described by the first canonical function is the strongest suggesting that all precollege mathematics measures are important and directly related to each other based on the shared signs and magnitudes of the structure coefficients. Student GPA and time to graduation were important measures in the engineering outcome variates and were logically inversely related, which is also important. We describe Function 1 as a calculus ready pathway to engineering suggesting that earning high scores on all precollege mathematics achievement measures leads to a higher GPA and shorter time to graduation, and this is perfectly aligned with the extant research base.

According to the relationship described in Function 1, the MPE was the most influential measure related to engineering success, followed by college admission exams (i.e., ACT and SAT) and then scores on either AP Calculus exam. Hence, this function best represents a more traditional mathematics path to engineering success. National test scores have been used most often as predictors of engineering success [12]. This result mirrors prior studies highlighting the utility of calculus readiness exams as good predictors of engineering retention [50]. National tests are popular due to their ease of attainment and their common measure. Furthermore, although racial inequities in AP course access abound, graduation outcomes of more than 140,000 STEM majors revealed that differential access to high school courses did not affect postsecondary STEM enrollment or degree attainment [51]. These findings add credence to a more precise and informed approach to engineering student recruitment efforts.

The MPE was designed by the Department of Mathematics of our university to predict calculus readiness for STEM majors, which is one plausible explanation for its relative importance to the relationship within and between the variates. Moreover, it is mandatory. This separates it from the AP Calculus exams, which are neither universally accessible nor require mandatory student reporting and prevents it from being upwardly biased. Thus, the MPE effectively helps in correctly placing students in an appropriate mathematics sequence by providing a measure that is better aligned with the specific university expectations for STEM success. In summary, we encourage relevant stakeholders to consider a calculus readiness rather than calculus proficient approach to the mathematics preparation of potential engineering students that accounts for the possibility of curricular misalignment between standardized mathematics exams and the specific objectives emphasized in respective STEM disciplines.

#### 6.2 Canonical Function 2 Interpretation

The relationship characterized in Function 2 represents an accelerated calculus approach. The relationship presented in the second canonical function is four times weaker but indicates that AP Calculus BC scores and time to graduation are the most influential factors across the two variates. This relationship can be characterized as "an accelerated approach." One explanation for this relationship is that there exists a group of very intelligent students who embark upon an accelerated path to graduation based on the completion of two semesters worth of calculus by excelling on the AP Calculus BC examination. They take AP courses, use the credit to forgo both semesters of calculus, and manage to pass all their classes without earning top scores; thus, GPA is no longer an influential measure in the model. This is only one of many explanations for this trend but based on the summary statistics provided in Table 1, this is definitely an important consideration.

There are two other plausible explanations for this phenomenon, both of which address the shortcomings of depending solely on student access to and success in AP courses to determine calculus readiness. Students who earn calculus AP credit will forfeit two valuable types of curricula by bypassing college calculus: (1) the implemented curriculum and (2) the hidden curriculum. These curricula are important because they are related to conceptual knowledge gaps as well as acclimation difficulty, which can manifest as a lack of belonging in engineering, both of which attribute to student attrition [52]. These challenges can create enduring issues for students throughout their academic careers.

The implemented curriculum is often different from the information presented in textbooks. For instance, many mathematics departments work in tandem with engineering departments to deliver calculus sequences that are tailored to address the learning outcomes most pertinent for success in engineering. Thus, by bypassing college calculus, they are missing exposure to explicit conceptual connections that are reflected in the implemented curriculum and the knowledge of which is necessary to understand the role of calculus within engineering. Therefore, allowing for the possibility of small gaps in these students' conceptual understanding of calculus knowledge that do not prevent them from finishing but causing some students to struggle mastering engineering concepts and earning high grades. This gradation in the level of calculus achievement weakens the strength of the relationship between precollege measures and GPA and time to graduation for this group of engineering students.

Relatedly, an alternative explanation is that earning high scores in AP calculus prevents some students from engaging in opportunities to learn about the elements of the "hidden curriculum," or the unique nuances related to the culture of studying engineering that are often instilled in freshman STEM courses. For example, many engineering colleges develop student cohorts that take the same courses in which they develop comradery forming the foundation of a problem solving community. Many students on an accelerated engineering pathway will effectively graduate, but their transcript and their ability to translate their knowledge into field-specific knowledge may be hindered by their lack of opportunity to participate in the community of learners.

#### 6.3 Limitations

Our findings support an emerging idea that deep mathematical understanding and fluency across mathematics disciplines were more important to academic success that relies on mathematics success. For example, using hierarchical linear models and controlling for student demographics and backgrounds, based on students' later performance in college calculus, mastery of the prerequisite mathematics for calculus (i.e., algebra, geometry, and precalculus) had a more positive effect on later academic success than taking a high school calculus course [53]. Our greatest contribution is that from our retrospective study we were able to examine the variables that appear in the vast majority of studies about engineering success and disaggregate individual and combined contributions and the implications for when and how certain variables were useful. However, all retrospective studies have important affordances and limitations. For example, retrospective studies can form the basis on which prospective studies are planned, however, they depend on extant data that were not originally designed to collect data for research. Because of this, some information is bound to be missing, selection biases may also affect the results and reasons for differences between groups and lost cases may lead to bias. Readers need to critically evaluate the methods and carefully interpret the results of retrospective studies.

### 7. Conclusions

Our work makes three unique contributions to the field. First, we developed a model to examine the relationship between precollege mathematics achievement and engineering learning outcomes by performing a CCA, a ground-breaking approach. Second, found that some prerequisite variables and their contribution to predicting post-secondary engineering success to be meaningful and important for future considerations. Third, because CCA is one of the most advanced methods in the General Linear Model, it afforded the ability to tease out the unique contributions of factors within the multivariate context to determine the importance of each of the commonly used variables. Our key findings is that high school calculus can help students with post-secondary success, but it is not sufficient for post-secondary engineering success. Contrary a popular belief that taking AP Calculus AB or BC and earning college credit is fortuitous, from our findings skipping the first two semesters of college calculus may actually result in lower grades in college and result in a longer time to graduation or opting out of a STEM major.

The relationship described by the first canonical function is the strongest correlation and suggests that all precollege mathematics measures are important and are directly related to each other. That is, their functioning was both necessary individually but also, and perhaps more importantly, provided a synergistic benefit when used in concert with the other variables. We also demonstrated that the MPE is the most influential measure in predicting engineering success, followed by college admission exams (i.e., ACT and SAT) and then scores on either of the AP Calculus exams. We encourage relevant stakeholders to consider a more efficient approach to assessing the mathematics preparation, specifically the calculus readiness, of potential engineering students that accounts for the possibility of curricular misalignment between standardized mathematics exams and the specific objectives emphasized in STEM disciplines.

The second canonical relationship, in contrast, indicates that AP Calculus BC scores and time to graduation are the only two variables of importance across the two variates and this relationship is negative. We posit as an explanation for this relationship that there exist a group of very intelligent students who embark upon an accelerated path to graduation by taking AP courses, using the credit to forgo both semesters of calculus, and managing to pass all their mathematics classes without earning top scores in any. As a result, GPA is no longer an influential measure in the model. If taking and passing their calculus classes with high grades, then grades too would have been important. We suggest that students who use their AP credit to skip taking calculus in college may lack some of the explicit conceptual connections necessary to understand the role of calculus within engineering and that are reflected in the implemented curriculum. As a result, there is the possibility that there are small gaps in these students' conceptual understandings of engineering-specific calculus knowledge that do not prevent them from finishing their degrees but do cause some of them to struggle unnecessarily and actually earn lower passing grades than students who take calculus in college.

Both models provide evidence for the idea that it is not necessary for students to take a calculus course while still in high school to succeed in engineering. Rather, it is important that they have a strong mathematics foundation that supports calculus readiness. This finding from our study extends findings from prior research on mathematics education, which indicates that high school calculus readiness can be more effective than high school calculus completion. Hence, we encourage stakeholders involved in preparing students to pursue engineering to possibly reevaluate their curriculum and advising recommendations for future engineering majors.

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