

Experimental Demonstration of a Fundamental Concept in Electromagnetics*

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Engineering and physics students usually have difficulty grasping that voltage in electrical circuits is actually a measure of energy. This paper describes an advanced experiment that was designed at the University of West Florida for directly demonstrating that the voltage between two given points in an electrostatic field is a measure of the energy required to move a unit charge between those points. In the experiment, a computerized vehicle transports a charged metal plate between two electrodes connected to a voltage source, while directly measuring the energy exerted in moving the plate. Assessment of the students' understanding of the concept after they perform the experiment has shown an improvement from about 10% (of the total number of students) to nearly 98%. Since the experiment helps the students perform actual measurements and verify a theoretical concept, its impact is more profound than otherwise teaching the concept through simulations.

Keywords: voltage-energy relationship; experiments in electromagnetism; potential difference; work in electrostatic fields

INTRODUCTION

THE POTENTIAL DIFFERENCE, or simply voltage between any two points in an electrostatic field is defined as the 'energy required to move a unit charge of one Coulomb between those points' [1, 2]. This fundamental concept is usually taught to engineering and physics students in introductory electricity and magnetism (E&M) courses. The concept is typically introduced to the students by means of an illustrative figure such as the one shown in Figure 1.

Experience in interacting with students, however, has shown that students usually have great difficulty grasping that concept. This, in turn, affects their understanding of related physical concepts, such as the concept of the 'electron volt', for example. Hence, entire subjects, such as solid-state physics, become inaccessible to the student who has had difficulty grasping that basic concept. To solve this problem, an advanced experiment was designed by the author for demonstrating the concept in Fig. 1 practically; that is, by moving a charge between two metal electrodes connected to a voltage source and directly measuring the energy exerted in moving the charge. The experiment was performed as part of the Electromagnetics I course at the University of West Florida. The majority of the students enrolled in the course are electrical engineering students, with a small number of physics students. Assessment of the students' understanding of the concept after they perform the experiment has shown an improvement from about 10% (of the approxi-

mately 140 students who conducted the experiment over a period of two years) to nearly 98%. In the past, a number of experimental and computational methods for teaching concepts in E&M were described in the literature [3, 4].

THE EXPERIMENT

General description

Fundamentally, the concept in Figure 1 can be demonstrated in a simple, low-tech manner by manually moving, for example, a charged metal plate between the two electrodes, while taking

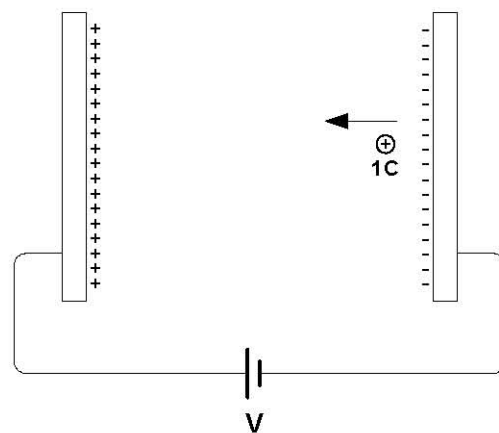


Fig. 1. A DC voltage V is applied between two electrodes. The energy in Joules that is required to move a positive charge of 1 Coulomb against the field from the negative electrode to the positive electrode is precisely equal to V .

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measurements of the force exerted on the plate by means of a balancing set of weights or by means of a spring mechanism. The energy exerted in moving the plate can then be calculated by numerical integration. This kind of experiment is described in [5]. Some educators also offer a simulation [6] for demonstrating the concept. Unfortunately, however, such crude methods for performing this fundamental experiment have shown to be inaccurate; in addition to failing to stimulate the student's interest. The advanced version of the experiment that was implemented at the University of West Florida is shown in Figure 2.

In the experiment, a computerized vehicle transports a charged metal plate between two electrodes that are connected to a source of high voltage. A highly accurate force meter provides a digital read-out of the force that acts on the plate, which is fed to a microprocessor board for processing. Simultaneously, the vehicle's speed is measured by a tachometer and digitized. The on-board microprocessor finally performs numerical integration and calculates the energy exerted in transporting the plate. In more detail: the motorized vehicle shown in Figure 2 measures approximately $50 \times 30 \times 20$ cm and carries a highly sensitive digital force meter on its roof. The force meter used is the Model BG Digital Force Gauge from Checkline, Inc. [7]. That meter has a resolution of 2×10^{-4} Newtons, and provides its digital output through an RS232 interface. The input pressure rod that is attached to the meter carries a Teflon mount, as shown, to which an aluminum plate that measures $30 \times 30 \times 0.2$ cm is attached. The force meter is insensitive to the vertical force (weight of the aluminum plate); it is sensitive only to the horizontal force. In this case,

the horizontal force is the electrostatic force that will be acting on the plate, in addition to the wind drag. Typically, the wind drag is negligible for speeds less than about 1 m/s [8]; and the vehicle's speed is adjusted to be less than 1 m/s.

As shown in Figure 2, the movable aluminum plate travels between two fixed aluminum plates of similar dimensions (both fixed plates are mounted on walls). The surfaces of both fixed plates are covered with a very thin sheet of Polystyrene (an excellent dielectric and insulator), so that no transfer of charge will occur between the fixed plates and the movable plate when the latter comes in contact with the fixed plates. Originally, the movable plate will be in contact with the first fixed plate shown in the figure, when the vehicle is at its home position. At that position, a small DC voltage of 12V is applied between the movable and the fixed plates so that the movable plate will acquire a charge. The student can calculate the charge Q on the plate using the relation:

$$Q = CV$$

where C is the capacitance between the two plates. The capacitance C is first calculated using the relation:

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

where ϵ_r is the dielectric constant (or relative permittivity) of the insulating Polystyrene layer, ϵ_0 is the permittivity of free space, A is the area of the plate, and d is the thickness of the dielectric layer. Because the dielectric constant of Polystyrene is only about 2.5, and because the contact

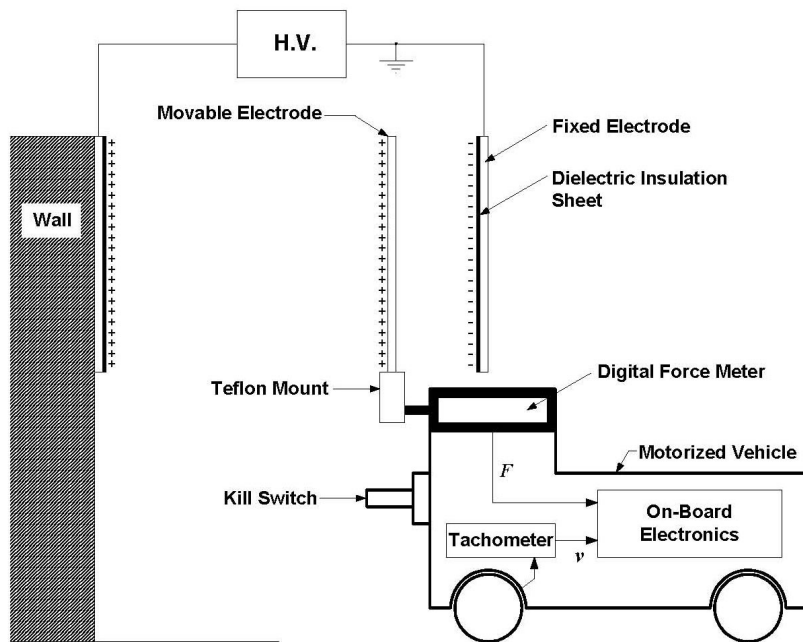


Fig. 2. Advanced experiment for demonstrating that voltage is a measure of energy.

between the movable plate and the fixed plate is never perfect (i.e. air gaps are usually present), the overall capacitance is usually found to be very nearly that of an air capacitor (i.e. $C \approx \epsilon_0 A/d$). Before the experiment is conducted, the student is asked to discharge the movable plate by touching it with the probe of a highly sensitive Keithley electrometer that is set in charge measurement mode, in order to verify that the charge on the plate conforms with the equations. The electrometer that was used is the Keithley model 610C [9], which is capable of measuring net charge with a resolution of 10^{-15} Coulombs. The student would then charge the plate one more time, and then disconnect the 12V battery. A high-voltage (HV) DC generator that is connected between the two fixed plates is then turned on (see Figure 2). The HV generator used in the experiment is the Glassman model WR100R2.5-11 [10], which generates voltage up to 125 kV. The extremely high voltage is needed to obtain an electric field intensity of sufficient magnitude between the fixed plates, in order to inflict a measurable force on the movable plate (the distance between the fixed plates is 1 m). The HV generator is set in a current-limited mode, so it does not present a danger more than that of the common Van de Graaff generator, found in many laboratories. In addition, to prevent any possibility of inadvertently touching the HV terminal that is not grounded and getting shocked, that HV terminal is very well insulated, and the plate connected to that terminal is completely covered with a sheet of Polystyrene, as indicated (see

Figure 2). A Start switch on the vehicle is finally pressed to set the vehicle in motion. It is to be noted that the strong electric field between the fixed plates will create a symmetrical and opposite distribution of charges on the moving plate; but the net charge Q on that plate will not be affected.

The energy exerted in moving the charged plate is given by

$$E = \int_0^X F dx$$

where F is the electrostatic force acting on the plate, x is the distance, and X is the total distance traveled. Although the force can be calculated from the theory of electrostatics, the objective of the experiment is to physically measure the force and integrate it along the path, in order to demonstrate that the value of the integral is simply equal to the applied voltage (note: the charge on the movable plate is actually a few micro-Coulombs, and hence the charge must be included as a scaling factor in the calculation; that is, the value of the integral will be equal to QV instead of V). Performing the above integration is difficult, unless the distance traveled can be measured directly and precisely. The integral, however, can be alternatively written as follows:

$$\begin{aligned} E &= \int_0^x F \frac{dx}{dt} dt \\ &= \int_0^T (Fv) dt \end{aligned}$$

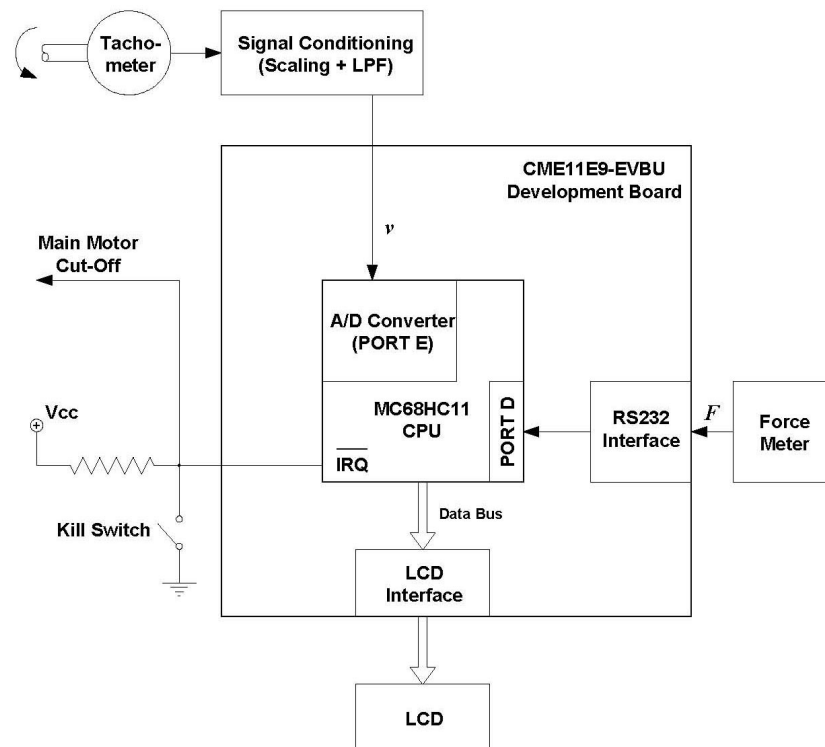


Fig. 3. Block diagram of electronic system carried by the vehicle.

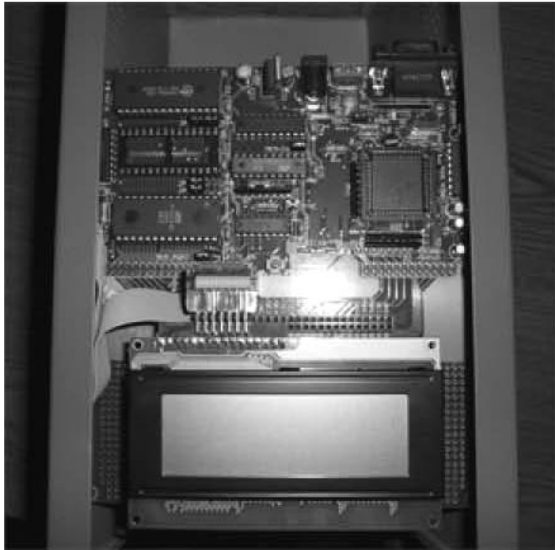


Fig. 4. MC68HC11 microprocessor board and LCD display mounted inside the vehicle.

where v is the velocity of the vehicle and T is the total travel time. Fortunately, the velocity can be measured precisely by means of a tachometer and a signal conditioning circuit. The circuitry in the vehicle must therefore measure and sample both the force and the speed of the vehicle simultaneously, take the product, store the results as a digital array, and finally perform numerical integration with time being the independent variable. All such functions are accomplished with a MC68HC11 microprocessor board that is carried on board the vehicle. As indicated above, the result of the above integral will be equal to QV , where Q is the charge on the moving plate. The student must therefore divide the final result displayed by the microprocessor board by the measured charge Q and verify that the ratio is indeed equal to the applied voltage.

The circuit used in the experiment

A block diagram of the electronic system used in the experiment is shown in Figure 3. A separate system is used to control the vehicle itself, and that system is not discussed in this paper.

As Figure 3 shows, the heart of the system is the CME11E9-EVBU microprocessor development board, manufactured by Axiom Technology, Inc. [11]. That board is based on the MC68HC11 microcontroller, and offers numerous features and interfaces that are suitable for this experiment. As shown in Figure 3, the signal from the tachometer (a voltage that is proportional to the speed of the vehicle) is first processed through a signal conditioning circuit consisting of a low-pass filter and a scaler. The signal is then directly fed to the analog-to-digital (A/D) converter of the microprocessor. The force measured by the force meter is supplied in digital form to the microprocessor through an RS232 interface. The assembly code

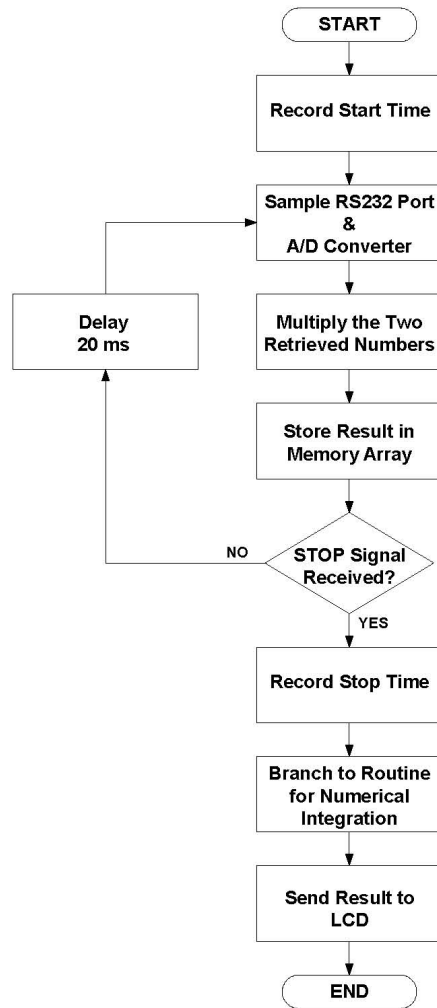


Fig. 5. Flow-chart of assembly code used to perform the experiment.

written for the task instructs the microprocessor to sample both signals at intervals of 20 ms (the delay needed for the A/D conversion is approximately 10 ms), multiply the sampled values, and store the product in a digital array on the on-board memory.

The vehicle is designed such that when the movable plate comes in contact with the remote fixed plate to within 1 mm, the “kill switch” shown in Figures 2 and 3 touches the wall and triggers. As the circuit diagram shows, this action results in an “active-low” signal that shuts the main motor of the vehicle off and simultaneously generates an Interrupt Request to the microprocessor. At this point the microprocessor records the total duration of the experiment, then jumps to a subroutine for performing numerical integration of Fv over time, according to the above integral. The result is finally transmitted to a liquid-crystal display (LCD) that is attached to the board. Figure 4 shows a photograph of the microprocessor board and the LCD mounted inside the vehicle.

A flowchart of the algorithm described above is shown in Figure 5. All the assembly code used fits on

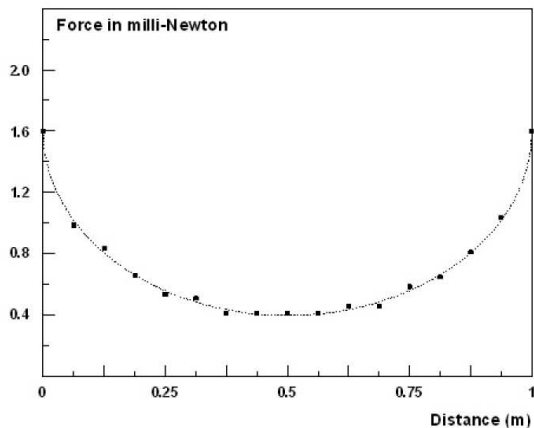


Fig. 6. Force measurements along the path of the vehicle (a smoothed least-squares best fit curve is shown as a dotted line).

the RAM that is available on the microprocessor board (Interested readers can contact the author who will be glad to supply the detailed code and the detailed schematics used in the apparatus.)

Numerical results

According to the classical theory of electrostatics, the force acting on the movable plate should be uniform and equal to QE (where E here is the electric field intensity), if the electric field between the two fixed plates has a uniform distribution. Figure 6 shows a plot of the force measurements that were taken along the path of the vehicle (0 to 1 meter).

As the figure shows, the force was not uniform along the path, which indicates a non-uniform electric field. This is not surprising, given that the distance between the two fixed plates is large by comparison with the size of the plates. A uniform electric field, however, is not necessary for accurately computing the energy exerted in moving a charge against the field (which must be equal to the potential difference regardless of the field distribution).

Figure 7 shows a histogram of the different values of the calculated voltage (or potential difference) for different runs of the experiment.

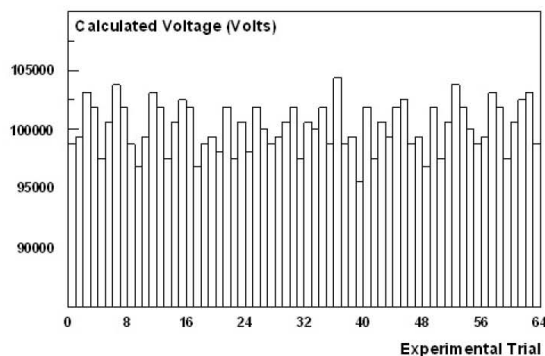


Fig. 7. Histogram of the values of the calculated potential difference for 64 different runs of the experiment.

The exact voltage applied was 100,000 volts (which was maintained by the sophisticated, closed loop controlled HV generator).

As the histogram shows, the maximum error in the computed value was approximately 5%. The error was found to be almost entirely due to an error in the initial calculation of the charge Q on the movable plate. The experiment was later modified by measuring the charge Q directly, instead of calculating it from Eq.(1). Given a more accurate estimation of Q , the error in the computed potential difference was reduced to a maximum of 1%.

STUDENTS' LEARNING AND UNDERSTANDING OF THE CONCEPT

A number of surveys and assessments were conducted over a period of two years to determine how the experiment affected the students' understanding of the physical concept. The study involved a group of 140 students in total. After the theoretical concept was first introduced, and before the experiment was conducted, the students were given a few quiz questions to determine whether:

- a) the student fundamentally understood the physical concept, or
- b) the student did not grasp the concept.

A typical quiz question to gauge the student's understanding of the concept is the following:

The label on a battery says "12 Volts". This means that:

- It takes 1 Joule of energy to move a charge of +12C from the negative electrode to the positive electrode.

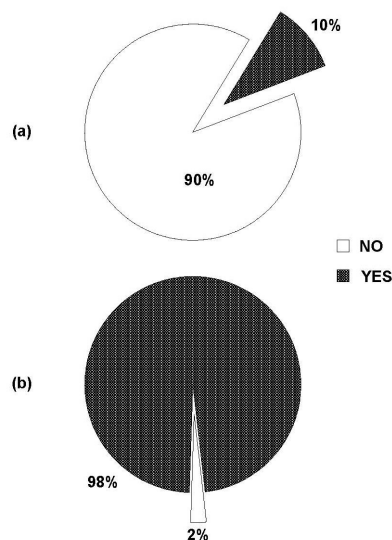


Fig. 8. Percentage of students who grasped the physical concept: (a) before the experiment was conducted; and (b) after the experiment was conducted.

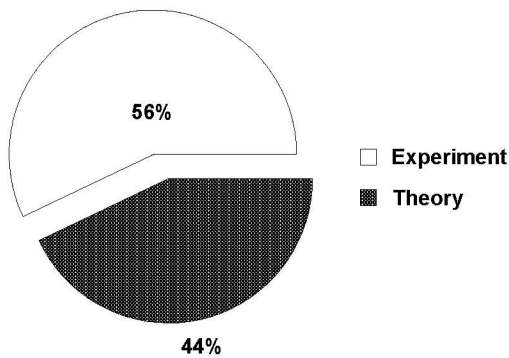


Fig. 9. The percentage of students who thought the experiment was more important for understanding the concept v. the percentage of students who thought that theory was more important.

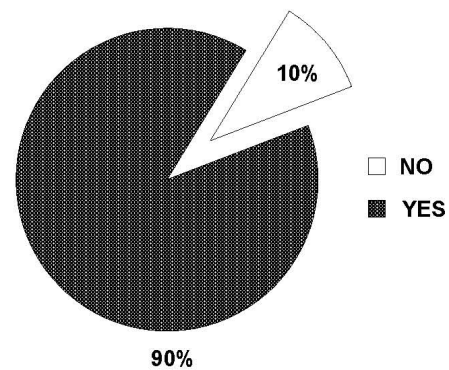


Fig. 10. Percentage of students who agree that the experiment captures their attention more than theory v. percentage of students who disagree.

- It takes 12 Joules of energy to move a charge of +1C from the negative electrode to the positive electrode.
- It takes 6 Joules of energy to move a charge of +2C from the negative electrode to the positive electrode.
- None of the above.

Another set of quiz questions were given to the students after the experiment was conducted. The results are shown in Figure 8.

As the results show, the percentage of students who fundamentally grasped the concept increased from 10% before the experiment to 98% thereafter. Two important surveys were then conducted. In the first survey, the students were asked the following question: which was more important to your understanding of the concept, the theoretical introduction or the experiment? The result is shown in Figure 9.

As the results show, the percentage of students who thought that the experiment is more important is only slightly higher than those who thought that the theoretical introduction was more important. In view of this result, together with the data in Figure 8, it is clear that the experiment contributed greatly to the students' understanding of the concept, but obviously did not eliminate the need for the formal introduction of the concept in the classroom.

In the second survey, the students were asked the following question: in general, do experiments that demonstrate fundamental scientific concepts

capture your attention more than theory? The result is shown in Figure 10.

CONCLUSIONS

As can be concluded from the surveys, advanced scientific experiments do contribute greatly to a student's understanding and grasp of the concepts involved, since they capture the attention of the vast majority of students more than theory (or simulations). Experiments, however, are not a substitute for the formal introduction of the theoretical concepts in the classroom, as the surveys showed.

The author, along with students at the University of West Florida, is currently working on enhancing the basic experiment; this includes:

- Using a much higher charge on the movable plate, and changing the shape of the plate from rectangular to circular in order to minimize charge leakage due to corona effects at the edges.
- Building a high-voltage powered, hand-held charge injector for injecting a precise amount of charge on the movable plate and for displaying it digitally before the experiment is conducted.
- Designing a digital force meter with higher sensitivity than what is available commercially. This will allow the use of lower voltages between the fixed electrodes in future experiments.

REFERENCES

1. W. H. Hayt and J. A. Buck, *Engineering Electromagnetics*, McGraw Hill, New York, NY, (2006).
2. D. K. Cheng, *Field and Wave Electromagnetics*, Addison Wesley, Reading, MA, (1992).
3. J. Faiz and M. Ojaghi, Instructive Review of Computation of Electric Fields using Numerical Techniques, *Int. J. Eng. Educ.*, **18**(3), (2002), pp. 344–346.
4. S. Ghosh, Electromagnetic Field Theory as a Basis for the Odd Parity Rule in Computational Geometry, *Int. J. Eng. Educ.*, **16**(1), (2000), pp. 68–72.
5. H. W. Fullbright, Simple and inexpensive teaching apparatus for absolute measurement of voltage, *Amer. J. Physics*, **61**(10), (1993), pp. 896–900.

6. http://phet.colorado.edu/simulations/sims.php?sim=Charges_and_Fields&order=asc&sort_by=contribution_type_desc&Simulations%5B%5D=Charges+and+Fields&Types%5B%5D=all&Levels%5B%5D=all
7. Checkline, Inc., *Model BG Digital Force Gauge*, www.checkline.com (2008).
8. J. D. Anderson, *Fundamentals of Aerodynamics*, McGraw Hill, New York, NY, (1984).
9. Keithley Instruments, Inc., www.keithley.com (2008).
10. Glassman High Voltage, Inc., www.glassmanhv.com (2008).
11. Axiom Manufacturing, Inc., www.axman.com (2008).

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