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Classroom Methods for Demonstrating, Modelling and Analysing the Viscoelasticity of Solids*

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This work presents some topics related to the teaching and learning of viscoelasticity in solids at the undergraduate level. It is recommended that the molecular and mechanistic aspects assigned to the viscoelastic behaviour of materials should be focused on, besides all the mathematical formalism, experimental techniques and applications. Some suggestions are made on laboratory practices and demonstrations, where the use of dynamic mechanical analysis is exemplified. It is suggested that active and cooperative learning techniques could be implemented in this subject, in order to obtain higher results. Finally, some cases are presented on pedagogical experiments and demonstrations that could be executed in the classroom.

Keywords: viscoelasticity; glass transition; learning techniques; dynamic mechanical analysis; polymers.

INTRODUCTION

VISCOELASTICITY IS a property observed essentially in all classes of materials, which will define their time-dependence mechanical and rheological performance. A perfect elastic solid responds instantaneously with a deformation under a static load, keeping the strain with time. Such a system stores all the energy utilised in the deformation induced by the developed stress, which is all used in an eventual recovery. Within the linear regime one should have $\sigma = E\epsilon$, where σ is the stress, E is the Young's modulus and ϵ is the strain. Conversely, in a perfect Newtonian liquid, the stress is proportional to the strain rate: $\sigma = \eta d\epsilon/dt$, where η is the viscosity. In this case, all the mechanical energy used in the deformation of a perfect viscous liquid is dissipated as heat, preventing the recovery of its shape. Most materials feature a hybrid behaviour called viscoelasticity [1, 2]. Lakes [3] develops these concepts and, in general, provides a good basis for developing a full curriculum on viscoelasticity in solids.

Understanding the viscoelastic properties of materials can be useful in the fabrication of objects/devices for specific applications—an interesting example is the foam earplug, specially designed to be highly pliable and able to fully recover its initial form, which allows a perfect fit of the foam cylinders into the ear canal, thus maximising the protection from excessive noise. Viscoelasticity is also the cause of the damping capability of materials, that is the ability to convert mechanical energy into thermal energy. The damping capacity of materials will depend on factors such as microstructure and composition, stress, temperature or frequency. Information at this level can be extremely useful in the development of systems for vibration dissipation (e.g. in industrial machinery and for protection of buildings or bridges from earthquake or winds), or in the optimisation of sports equipment, biomedical devices or monitoring instruments, musical instruments, and impact absorber concepts. Understanding the solid-state rheological properties is important in many engineering problems. Knowledge of viscoelastic properties can also be useful for understanding the particular state of a material (rubber, glass, etc.) and to gain information about molecular/morphological factors, such as nano/micro-structural features, crystallinity, anisotropy or, in the case of polymers, branching, cross-linking and a variety of other architectural aspects of their structures.

The subject of viscoelasticity in solids, with different depths of development, has been included in a variety of undergraduate curricula, such as Materials/Polymer/Mechanical/Civil/Biomedical Engineering, or even in the basic sciences (Biology or Physics). However, the method of presenting the concepts and the practical examples used should be adequate to the tools ultimately needed by the students at the end of their education. This work presents some reflections in this context. It should be stressed that viscoelasticity, reflected as macroscopic manifestations, should be linked to the structure of the dynamics of materials, and thus should be understood according to a molecular

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basis. Some considerations on the most important topics that students should be familiar with are presented, together with a list of relevant references. Alternative learning strategies are presented, besides the conventional lecture format (overhead slides and class handouts) and typical demonstration laboratory practices. Simple experiments and multimedia strategies enhance the interactivity and help solidify the theoretical concepts. Active and cooperative learning can also be adequately implemented as effective pedagogical techniques. Viscoelasticity topics can also be implemented in learning methods based on projects or in integrated laboratories. Laboratory practices should be designed according to the student's profile. An example will be shown on the use of Dynamic Mechanical Analysis (DMA) as a suitable technique for accessing the viscoelastic properties of solid materials.

LABORATORY PRACTICES: THE USE OF DMA

Different equipment and techniques may be used in laboratory experiments to assess the solid viscoelastic properties of materials. A detailed analysis can be found elsewhere [1–5]. I believe that dynamic mechanical analysis, DMA, is the most suitable technique for such demonstrations. Commercial DMA equipment is usually very versatile and can perform different kinds of experiments, including transient tests (e.g. creep and stress relaxation) at constant or varying temperatures. DMA experiments may be performed using most materials, but they are especially suitable for investigating polymers [6].

Summarising the information that can be extracted from such experiments, when a specimen is subjected to a sinusoidal load, with a rate defined by a frequency f (in cycles.s^{-1}, or Hz) or an angular frequency $\omega = 2\pi f$ (in rad.s^{-1}), the response (a strain), though sinusoidal, is not exactly in step with the developed stress and will lag behind the stress by a phase angle δ between 0° and 90° (Fig. 1). The sinusoidal stress, σ, and strain, ϵ, written in a complex notation are given by:

$$\sigma = \sigma_0 \exp(i\omega t + \delta) \quad \text{and} \quad \epsilon = \epsilon_0 \exp(i\omega t) \qquad (1)$$

where σ_0 and ϵ_0 are the stress and strain amplitudes and i=$(-1)^{1/2}$. A full description within this linear region may be provided by the complex modulus, $E^*(\omega)$, or the complex compliance, $D^*(\omega)$, defined, respectively, as:

$$\begin{aligned} E^*(\omega) &= \frac{\sigma}{\epsilon} = (\sigma_0/\epsilon_0)\exp(i\delta) \\ &= (\sigma_0/\epsilon_0)(\cos\delta + i\sin\delta) = E' + iE'' \qquad (2a) \end{aligned}$$

$$\begin{aligned} D^*(\omega) &= \frac{1}{E^*} = \frac{\epsilon}{\sigma} = (\epsilon_0/\sigma_0) \\ &(\cos\delta - i\sin\delta) = D' + iD'' \qquad (2b) \end{aligned}$$

The storage modulus, E', is the elastic, or the real component of E^*, which is in phase with σ. The storage modulus is related to the stiffness of the material. The loss modulus, E', is the viscous (also called imaginary) component of E^*, which is $\pi/2$ out of phase in relation to σ. E' is associated with the dissipation of energy, as heat, due to internal friction at the molecular level. It is essential to derive the relationship between the dissipation of energy in a complete cycle, per unit of volume, E_{dis}, (and the maximum stored energy, $E_{st,\max}$) and E' (and E'). This will demonstrate to the student a clear link between such basic considerations and the practical implications. The ratio of $E_{dis}/E_{st,\max} = 2\pi \tan\delta$ is often a measure of the damping capability of a material and is referred to as the specific loss or specific damping capacity.

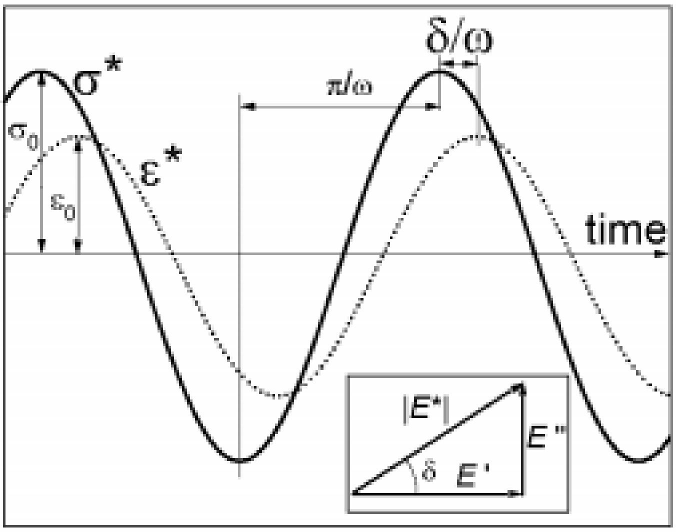


Fig. 1. Scheme of the time domain mechanical response (strain, ϵ) of a viscoelastic material subjected to a sinusoidal stress (σ).

Experiments may take some time, as scanning rates are relatively slow. Therefore, practical exercises and data treatment (calculation of the glass transition temperature, construction of master curves, adjusting results to physical models) or simple questionnaires, containing basic aspects of the theoretical concepts associated with the experiments and on the general information that can be extracted from the results, should be distributed in order to enhance the outcome of the class. Previous results or complementary material (e.g. differential scanning calorimetry results and morphology characterisation of the materials that are being analysed) can also be discussed while the DMA experiments are running. During the experiments it is useful to consider the calibration procedures (temperature, force, displacement, etc.) and to reflect on the choice of the optimum experimental conditions that should be used. The time can also be used to discuss the link between the viscoelastic features of the materials and the associated motion at the molecular level.

The modelling of the data is very important, as it provides a physical interpretation of the experimental results and allows prediction of behaviour in other situations. Simple combination of pure elastic (springs) and viscous (dashpots) materials may be useful to visualise how the viscoelastic behaviour can arise. The simplest ones are the Maxwell model (spring and dashpot in series) and the Voigt model (the two elements in parallel). However, the more realistic model, which may describe both creep and stress relaxation is the *standard linear solid* (Fig. 2a).

It can be shown, being a good example to be given in a lecturing class, that the strain and stress in this model can be generically related by:

$$\frac{d\epsilon}{dt}(E_1 + E_2) + \epsilon\frac{E_2}{\tau} = \frac{\sigma}{\tau} + \frac{d\sigma}{dt} \tag{3}$$

where $\tau = \eta/E_1$. The concept of a constitutive equation can be explained using this equation as an example, which can be easily adapted to creep or stress relaxation situations. From the time dependence of σ and ϵ in Equation (1), the two components of the complex modulus can be obtained, as a function of frequency:

$$E'(\omega) = E_2 + E_1\frac{\omega^2\tau^2}{1+\omega^2\tau^2},\ E''(\omega) = E_1\frac{\omega\tau}{1+\omega^2\tau^2} \tag{4}$$

The section on pedagogical experiments will suggest the use of Microsoft® Excel worksheets to project graphics derived from equations in the classroom; the parameters are easily changed and help students to gain more insight into the phenomenology associated with viscoelasticity. An example is given in Fig. 2b, where typical curves of D', D', E', E' and tan δ are given as a function of log frequency. It must be stressed in the class that the solid linear standard is a rather simple model and that almost no material could be described according to that model. This will introduce the notion of the distribution of characteristic times and all the formalisms associated with the relaxation/retardation spectra in the description of the different quantities, as well as the inter-relationships between them. Naturally, the extent of this analysis will depend upon the nature of the course, the mathematical background of the students and the time available.

Many materials could be studied. Polymers are very suitable for demonstrations, as near the glass transition temperature great changes are observed upon the relevant viscoelastic quantities [1, 2, 6]. Fig. 3 shows an example of a DMA spectrum obtained from a poly(ethylene therephthalate) sample, PET, initially amorphous. This is a suitable material for demonstration activities and can

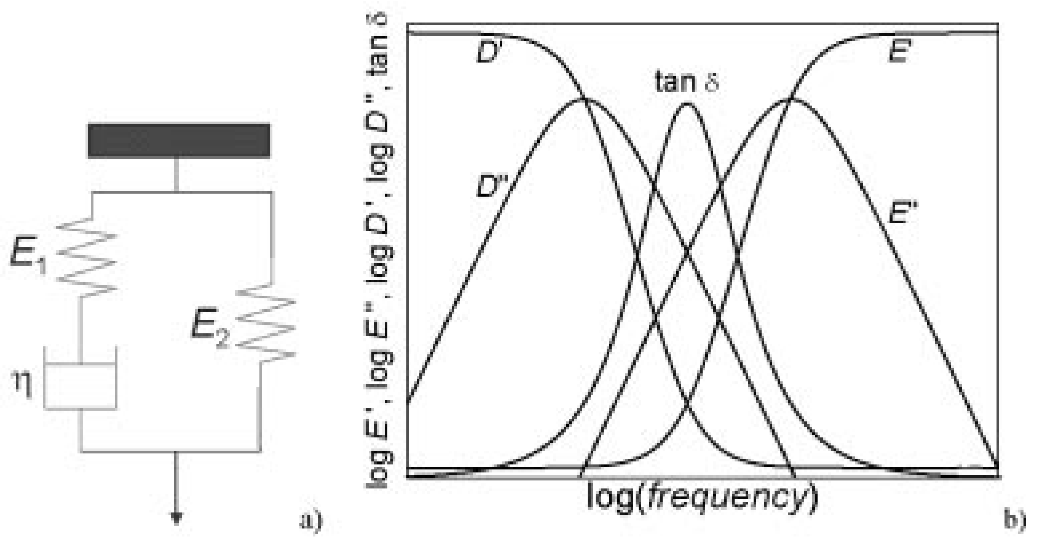


Fig. 2. The standard linear solid model (a) can be used to generate the frequency dependence of the most important viscoleastic parameters (b), using equations such as Equation (4).

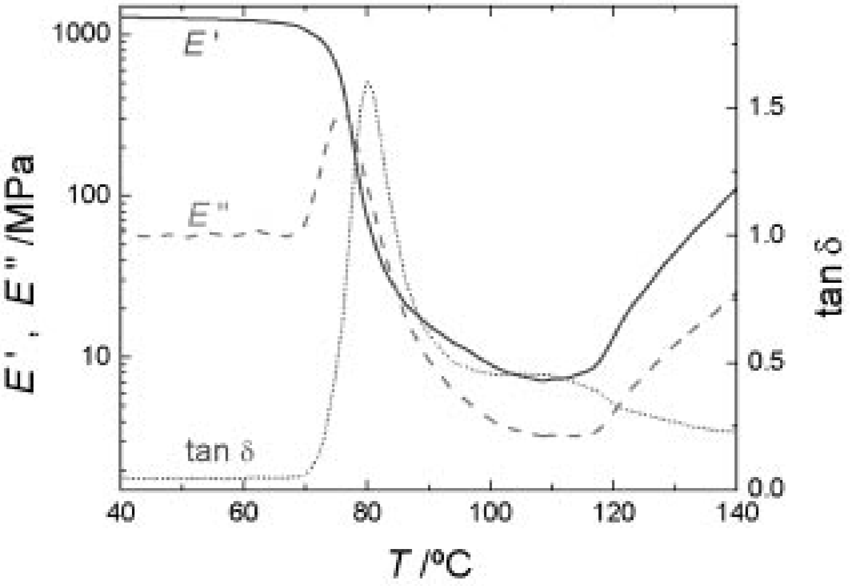


Fig. 3. DMA spectrum on poly(ethylene therephthalate), obtained for a frequency of 1 Hz. More details on the experimental procedure can be found elsewhere [7].

be obtained from plastic bottles. More details on the experimental conditions may be found elsewhere [7], as well as a more in-depth discussion of the results. In summary, the glass transition process may be identified by the peaks in E' and tan δ, associated with the cooperative motions of the polymeric chains, together with a pronounced decrease in E'. The association of DMA results with data obtained by differential scanning calorimetry could be beneficial; thermal properties using this technique with PET were discussed in another paper, also with an educational perspective [8]. An interesting point here is that above the glass transition temperature the material is able to crystallise (cold-crystallisation process). Such an event is responsible for the increase of E' (the material becomes stiffer) and of elasticity (tan δ decrease) that occurs above 115°C in this particular case. Interesting demonstrations can also be performed using polymer blends to investigate the miscibility of the components or polymer composites, where the effect of the fibre orientation or amount of content may be explored.

Laboratory experiments and demonstrations can be employed in a course focused on mechanical/ viscoelastic properties of materials, or rheology. However, experimental works can be designed for more transversal courses, such as integrated laboratories, or in project-based learning strategies.

Although laboratory classes are important, this work will be more focused on generic learning techniques, and the last sections will be devoted to improving the teaching and learning of viscoelasticity during lectures.

ACTIVE AND COOPERATIVE LEARNING

There has been a considerable amount of discussion about teaching techniques in the science education literature. As described by Paulson [9], a useful text including valuable references, these strategies include cooperative learning, classroom assessment, inquiry learning and active learning. All of these techniques can be implemented in the field of learning viscoelasticity, where small adaptations may be necessary depending on the nature of the course. As mentioned by Towns [10], cooperative (and active) learning leads to higher achievement, increased positive attitudes towards the subject area studied, higher self-esteem, greater acceptance of differences among peers, and enhanced conceptual development in a wide range of settings and across content areas. Some examples will be given here in this context.

Group activities

An important message to pass on to students is that it is highly probable that they will work in multidisciplinary teams. Growing fields such as biotechnology, biomedical engineering and materials science require teams of people with different areas of technical expertise. Therefore, it is important for students at the undergraduate level to work and learn effectively with other students. When forming groups, it is preferable to join heterogeneous students, in terms of profile, marks in previous disciplines, and gender. Each group should discuss their own strategy, what will be the role of each member in the group, and the advantages (and disadvantages) of working in a group. Individual conclusions can then be discussed between all the groups. The group members should recognise their individual skills and they should discuss their position in the group (manager, spokesperson, recorder, etc.).

Group activities can include graded homework problems that contain simple questions or more elaborate inquiries (for example, the deduction of Equation (7)—see below) that require much more effort. Group classwork can also be given to the students. A period of time is given for discussion within the group, and then a discussion can be initiated between the groups, where each group should justify its answer. Topics could include the different ways of solving problems or the different possibilities of measuring the viscoelastic properties relevant to solving a particular problem. In the next section, a specific example is given that can be included among group activities.

Discussion of scientific papers

For upper-level students with a more scientific profile (Physics or Materials Science), it is interesting to provide review papers to be analysed by groups of students. Such methodology was adopted in other fields, such as biochemistry [11], where Hodges pointed out the existence of three phases of discussion: (i) overview, where some questions are provided by the instructor, covering the key ideas of the paper and helping in providing the concepts and the vocabulary necessary to understand its content; (ii) background, where the students must research information on their individual topics and share it with their classmates through short informal presentations; and (iii) discussion, being the class-wide discussion of the article. Two students are designated as discussion leaders and should provide a more formal presentation; however, all students are required to write a short text summarising the content of the article. It was concluded that this method relies on students explaining concepts to others, a process requiring higher-level reasoning, as in other collaborative or cooperative learning strategies.

In the case of viscoelasticity, papers could go from very basic matters, such as the molecular origin of the anelastic properties of materials (see for example [12], for its relation with molecular motions in polymers), to measurement techniques [5], up to applications in specific areas, such as food [13]; of course, many more examples could be given here.

Concept tests

Specifically developed in chemical and physical education, the use of concept tests is a good example of active learning methods [14–16]. Briefly, conceptual questions are posed at appropriate times during the lecture along with a few possible answers; here, overhead transparencies or multimedia can be used. Students vote on the possible answers, then try to persuade their neighbours in the lecture room that they are correct, and finally vote again. Occasionally, the instructor may ask a student to provide an explanation for the correct answer. This provides the lecturer with online feedback about student learning. Moreover, the discussions allow the students to teach each other, and are thus a form of informal cooperative learning, where students may enhance their reasoning capability and communication skills.

In the case of viscoelasticity learning, questions can range from simple questions about elementary subjects to interpretation of experimental results or anticipated results in real situations. An example could be the use of Equation (3) to deduce the stress–relaxation behaviour of a standard linear solid, where different possibilities would be offered; or to find, among different creep/recovery curves, where a plastic behaviour could be observed, and how the viscosity associated with irrecoverable flow could be estimated (an example showing a creep curve exhibiting elastic, viscoelastic and plastic deformations will be shown later).

Finger signals

Another example of an active learning technique, similar to the idea of concept tests, is 'finger signals', a technique which has been implemented, for example, by Paulson in Organic Chemistry classes [9]. It is a very efficient way of interrupting a section of a lecture (before moving on to the next topic), enabling a dynamic break for the students. Again, a question is presented during the presentation of any concept during a lecture, with different numbered alternative answers. After a period of reflection, the students are asked to show their answer by holding the appropriate number of fingers against their chest, a strategy for making it difficult for students to accurately infer the answer of their colleagues. An initial idea of how much of the lecture has been understood by the students can immediately be obtained by the instructor. This can also offer the opportunity to ask the students (preferably the ones that answered in a different way) to present and defend their opinion, and short discussion periods can be opened up. This will again allow the student to share ideas and arguments, providing feedback and teaching each other. The instructor should act here as a facilitator, encouraging the discussion of ideas, rather than providing information.

PEDAGOGICAL EXPERIMENTS AND DEMONSTRATIONS DURING LECTURES

It is not effective to continually lecture for 50 minutes, presenting concepts and demonstrations. It has been shown that after 15–20 minutes of uninterrupted lecture, the retention and understanding of material drops rapidly [17]. In the previous section, it was seen that the (individual or group) response to questions may be an efficient way of monitoring the understanding of the students and reinforcing their knowledge of the topics. This will help also as an effective break between different subjects. Another possibility is to present some simple experiments and demonstrations during the class. Many ideas could be implemented in this context, but only a few will be presented here.

Anelasticity in rubber balls

To demonstrate the different viscoelastic properties of materials, there are commercially available

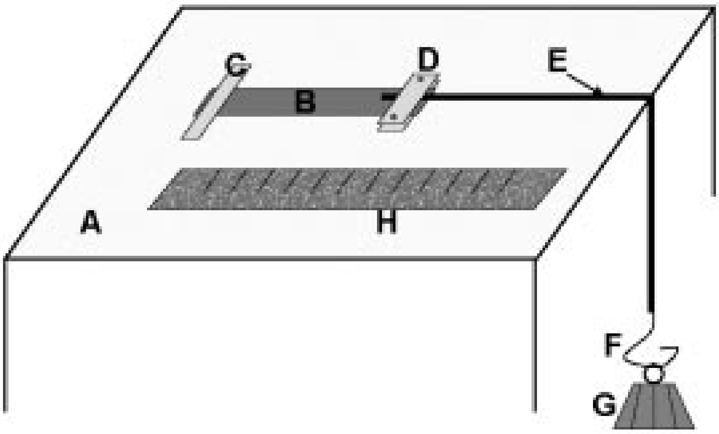


Fig. 4. Scheme of a set-up used to demonstrate creep and recovery in a classroom using an overhead projector. A: base of the projector; B: rectangular sample; C: adhesive tape to fix the sample at one end; D: clamp, to link the sample to the thread (E); F: stiff metallic hook, where a weight (G) can be suspended; H: transparent rule, which can be used to estimate the time-dependent length of the sample during the experiment.

rubber balls with different bounce capabilities. For example, I have been using balls of neoprene rubber and polynorbornene rubber, from Educational Innovations, Inc. (Norwalk, CT) with the name of Choositz Decision balls. The two balls are dropped from the same height onto a hard surface. The first ball behaves almost elastically, and bounces back to almost the same height as dropped. The polynorbornene rubber ball exhibits a much higher damping capability at room temperature, and after hitting the ground will demonstrate little bounce. For a more deep understanding the behaviour of the two materials, DMA experiments can be conducted (or the results shown in the class) in order to analyse the temperature dependence of E' and tan δ, at different frequencies. This will provide an example of relating basic viscoelastic properties of materials and their real behaviour.

Earplugs

Earplugs are cylindrical-shaped pieces of foam, somewhat larger than the ear canal. They may recover from 40% to 60% compression in 1 to 60 seconds and possess an equilibrium stiffness at 40% compression of 1.4 to 9 kPa [3]. A simple

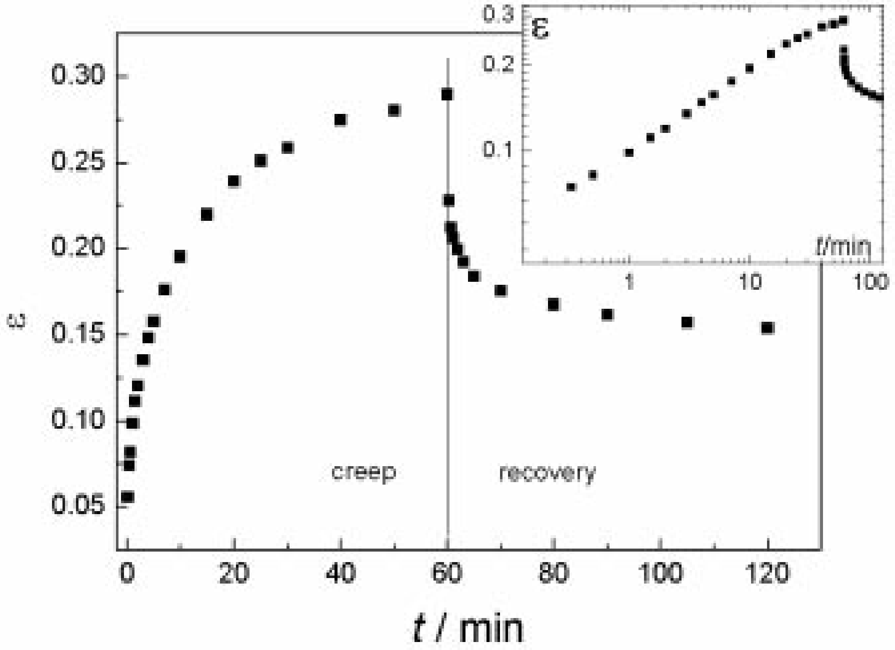


Fig. 5. Creep/recovery experiments performed in a sample extracted from a polyethylene bag with 243 mm length, 6.0 mm width and 0.02 mm thickness. An experimental set-up similar to the one described in Fig. 4 was used. A load of 185 g was imposed at time zero, corresponding to a constant stress level of 1.8 N, where the cross-section area of the sample does not vary during the experiment. After 60 min, the load was decreased to a much smaller value (30 g); this weight will allow the sample to stay tight. The recovery is monitored for another 60 min. The strain is calculated by measuring the chain's length, Δl, $\epsilon(t) = \Delta l / l_0$, where l_0 is the initial length of the sample. Δl was measured with a digital pachymeter, which enhanced the resolution of the measurement with respect to the conventional ruler. The inset graphic shows the same result but the axis is in a logarithmic form.

experiment can be done using an overhead projector. The deformed earplug is placed in the base of the projector at time zero, near a transparent rule. The evolution of the recovery is followed by monitoring the change of the projected shadow of the earplug. A rough estimate of the lengths can be made using a rule. It may be interesting here to comment on the errors committed in taking such measurements. Again, it is instructive to show some viscoelastic data, such as DMA results, and accurate creep/recovery measurements recorded in the laboratory.

Creep behaviour of polyethylene

A similar experiment to the last one can be performed to demonstrate creep (see scheme in Fig. 4). A rectangular film of polyethylene (B) is cut from a plastic bag and clamped at one extremity on the base of an overhead projector, A (a good adhesive tape can be used—C). The geometry of the sample should be measured using a pachymeter. The other extremity of the sample is clamped to a flexible thread (E) that is linked at the other end to a stiff metallic hook (F), where a weight can be suspended vertically (G). At time zero, the weight is placed and the deformation may be seen in the projected image of the sample. An estimate of the sample's length can be obtained by placing a transparent rule next to the sample. Different weights can be placed to analyse the effect of the load on the creep behaviour. The recovery of the sample may also be followed by taking out the weight from the support. For release times much longer than the creep time, a discussion on the unrecovered strain may take place. The creep data should then be analysed, including the plastic deformation that also occurs.

In Fig. 5, an example is shown where the creep and recovery of a polyethylene film is monitored during 120 min. The results clearly show the initial instantaneous elastic response, followed by a time-dependent deformation. The recovery also presents an immediate elastic response and a slow recovery, indicating that the creep response contained some viscoelastic character. However, it is clear that the initial dimension of the sample cannot fully recover, indicating that some plastic deformation occurred during creep. Instructive experiments consist of repeating similar creep/recovery tests with different creep loads, in order that several aspects can be investigated, such as the linear viscoelastic behaviour and the dependence of the creep load on the permanent strain after recovery. More sophisticated models than the standard linear solids (Fig. 2a) could be used to model the results. It is important here to reinforce the idea that the molecular motions involved in the time-dependent response should be described by a continuous distribution of characteristic times. Log-log plots are sometimes relevant to observe the data in another perspective. The inset graphic of Fig. 6 contains the same results, but in this case a quasi-linear relationship between log ϵ and log time may be seen, suggesting a power law between ϵ and time.

Creep experiments can also be performed using commercial DMA equipment. This allows for more sophisticated experiments, including tests at different temperatures. Moreover, complementary dynamic experiments can be usefully compared with transient results. A practical example, published elsewhere [18], that may inspire other works is the viscoelastic characterisation of commercially available fibre sutures of poly(vinylidene fluoride) using both creep and DMA tests.

It is important that students realise that, in most cases, a material is not subject to the same mechanical load (or strain), as it may change with time. Therefore the concept of creep tests should be extended, and the Boltzmann superposition principle should be introduced in this context. This principle states that, in a multi-stress situation, each loading step makes an independent contribution to the final deformation, which is obtained cumulatively from all contributions. If changes of stress occur at different times, $\tau_1, \tau_2 \ldots, \tau_i \ldots,$ then

$$\epsilon(t) = \sigma_0 D(t) + \Delta\sigma_1 D(t-\tau_1) + \Delta\sigma_2 D(t-\tau_2) + \ldots \Delta\sigma_i D(t-\tau_i) + \ldots \quad (5)$$

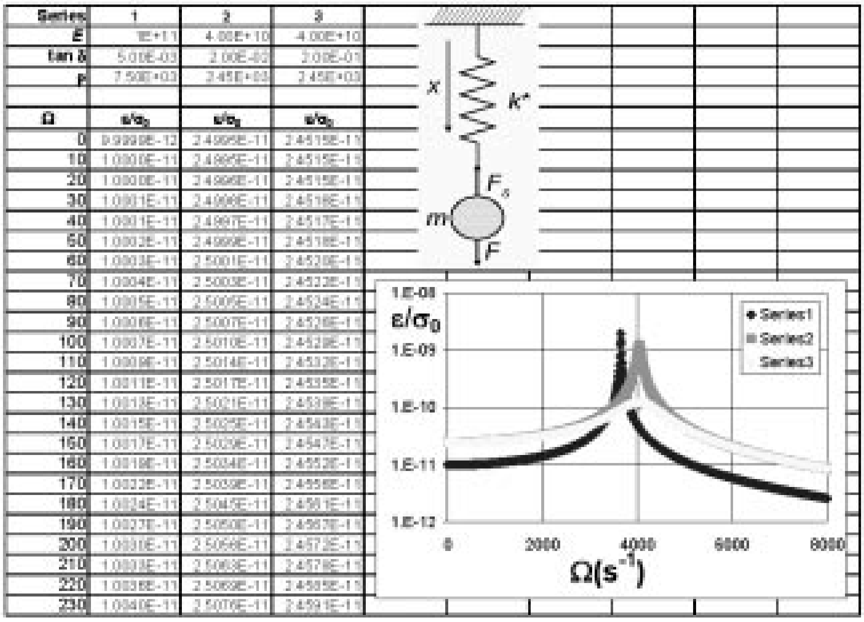
where $\Delta\sigma_i$ are the step changes of stress at time τ_i (i.e. $\Delta\sigma_i = \sigma_i - \sigma_{i-1}$). This equation holds for the simplest case of linear viscoelastic response. The Boltzmann superposition principle states that the response of a material to a given load is independent of the material response to any previous load histories, and thus the effect of a compound cause is the sum of the effects of the individual causes. Equation (5) may also be used to model the recovery process that corresponds to the removal of the creep stress at a certain time t_{rec}; in this case one should have, at a given time, $t > t_{rec}$, $\epsilon(t) = \sigma_0 D(t) - \sigma_0 D(t - t_{rec})$. This expression may be easily linked with the creep/recovery experiment on polyethylene (Fig. 5). Moreover, the presentation of Equation (5) may be facilitated by putting different weights on the polyethylene creep experiment at different time points. Considering that at $t = 0$ the specimen is not affected by its previous stress history, for more continuous and complex stress variation one may generalise Equation (5) to:

$$\epsilon(t) = \int_0^t D(t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau \quad (6)$$

It is essential that this principle be understood by the students, and examples should be provided according to the profile of the course.

Viscoelastic properties of bone

Especially in Biomedical Engineering undergraduate education, a similar creep experiment may be performed using biological tissue. A good example is bone, using a three-point bending



Series	1	2	3
E	1E+11	4.00E+10	4.00E+10
tan δ	5.00E-03	2.00E-02	2.00E-01
ρ	7.50E+03	2.45E+03	2.45E+03
Ω	ε/σ$_0$	ε/σ$_0$	ε/σ$_0$
0	9.9999E-12	2.4995E-11	2.4515E-11
10	1.0000E-11	2.4995E-11	2.4515E-11
20	1.0000E-11	2.4996E-11	2.4515E-11
30	1.0001E-11	2.4996E-11	2.4516E-11
40	1.0001E-11	2.4997E-11	2.4517E-11
50	1.0002E-11	2.4999E-11	2.4518E-11
60	1.0003E-11	2.5001E-11	2.4520E-11
70	1.0004E-11	2.5003E-11	2.4522E-11
80	1.0005E-11	2.5005E-11	2.4524E-11
90	1.0006E-11	2.5007E-11	2.4526E-11
100	1.0007E-11	2.5010E-11	2.4529E-11
110	1.0009E-11	2.5014E-11	2.4532E-11
120	1.0011E-11	2.5017E-11	2.4535E-11
130	1.0013E-11	2.5021E-11	2.4539E-11
140	1.0015E-11	2.5025E-11	2.4543E-11
150	1.0017E-11	2.5029E-11	2.4547E-11
160	1.0019E-11	2.5034E-11	2.4552E-11
170	1.0022E-11	2.5039E-11	2.4556E-11
180	1.0024E-11	2.5045E-11	2.4561E-11
190	1.0027E-11	2.5050E-11	2.4567E-11
200	1.0031E-11	2.5056E-11	2.4573E-11
210	1.0033E-11	2.5063E-11	2.4578E-11
220	1.0036E-11	2.5069E-11	2.4585E-11
230	1.0040E-11	2.5076E-11	2.4591E-11

Fig. 6. Section of a Microsoft® Excel worksheet where an application of Equation (7) is presented for three materials (Series 1 to 3). The inset graphic shows the normalised amplitude of a mass subjected to a periodic force with a circular frequency Ω in the steady regime. The inset scheme shows the damped oscillator model used.

mode; thus the set-up shown in Fig. 5 should be slightly changed. A good source of fresh bone can be obtained from chicken wings, readily available in any supermarket. The results should be compared with creep/recovery and DMA data obtained from the laboratory, that have been recently reported [19].

The damped oscillator

Mechanical vibration is a problem which may influence the precision of a machine tool or the performance of an electronic device, and different engineering strategies have been developed in order to minimise its effect. Vibration can also generate fatigue of materials or simply noise pollution. A good way to reduce mechanical vibration is to use damping materials [20] that possess the intrinsic ability to damp vibration. The simplest rheological model can be used in this context to describe the dynamic behaviour of a material that will be the damped oscillator (see scheme in the inset of Fig. 6), where a mass of a material with density ρ is hung on an anelastic spring (with a complex spring constant k^*). Various textbooks develop this model, such as the one by Schaller [20]. As mentioned before, an enlightening project would be to analyse this problem and learn how to achieve the dependence of the amplitude, ϵ, with the circular frequency Ω of the periodic force, F, that the mass is externally subjected to. The concept of resonance should be clearly understood. In the steady regime, $\epsilon(\Omega)$ is given by:

$$\epsilon(\Omega) = \frac{\sigma_0}{\sqrt{\left(\frac{E}{\rho}\tan\delta\right)^2 + \left(\frac{E}{\rho} - \Omega^2\right)^2}} \frac{1}{\rho} \tag{7}$$

where σ_0 is the amplitude of the applied stress and E is the elastic modulus of the spring. It is quite easy to implement this equation, for example, in a Microsoft® Excel worksheet. An example is given in Fig. 6, where Equation (7) is used to compare the dynamic behaviour of different damping materials.

This worksheet could be studied using a multimedia data projection, in order to explore with the students the effect of the relevant parameters, E, tan δ and ρ on the damping capability of a material. In this case, three materials are directly compared, but more can be included. One advantage of this demonstration is that, for a particular series in the worksheet, any of the three parameters can be changed, which will allow students to perceive immediately the effect on the normalised $\epsilon(\Omega)$ curve.

The first two series correspond to two high damping materials used in mechanical engineering (machine frame): grey cast iron and polymer concrete, also analysed by Schaller [20]. It can be seen that, despite the higher damping capability of the second material (higher tan δ), the deformation amplitude is higher than for grey cast iron (of course, when the perturbation is suppressed, the oscillation will decrease more rapidly in the case of the polymer concrete). Such behaviour can be interpreted, at low frequencies, by the difference in the elastic moduli ($\epsilon \sim \sigma_0/E$) and, at higher frequencies, by the difference in the densities ($\epsilon \sim \sigma_0/(\rho\Omega^2)$) [20]. This example clearly demonstrates that the loss factor is not the sole factor that will determine the ability of a system to absorb mechanical vibrations (a good compromise should exist between the three parameters), but it will be relevant, especially near the resonance frequencies.

Series 3 in the example of Fig. 6 is similar to Series 2, but with a tan δ ten times higher; it can be clearly seen that, at intermediate frequencies, the oscillation is highly reduced.

SUMMARY

Viscoelasticity is a topic that is currently addressed in different curricula. It was shown that, besides conventional lectures, viscoelasticity of solids could be introduced using a variety of learning strategies. Dynamic mechanical analysis is an adequate tool for characterising the solid-state rheological properties of materials, and can help in demonstrating several aspects of the viscoelastic features of polymers. Several examples of active and cooperative learning were presented as effective schemes to improve the learning of this subject. Finally, some pedagogical experiments were presented that could be implemented in the classroom. A special focus was given to creep/recovery tests on polyethylene films using an overhead projector and other basic objects. Moreover, suggestions were made for using multimedia tools to promote a more dynamic exemplification of the result of viscoelastic-derived equations.

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