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Designing a Buckling Experiment: a Teaching Experience*

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A laboratory demonstration of column buckling is important to civil engineering students. An experimental set-up consisting of a laterally loaded slender column subjected to an increasing axial load proved to be helpful in demonstrating buckling mechanisms. However, the interpretation of the student results had to be adjusted to take into account the actual support conditions, thus leading to improvements in the design, formulation and understanding of this laboratory experiment.

EDUCATIONAL SUMMARY

1. The paper describes new training tools or laboratory concepts/instrumentation/experiments in: laboratory concepts/experiment in column buckling and deformation under axial loads.
2. The paper describes new equipment useful in courses on mechanics of solids.
3. Undergraduate level students are involved in the use of the equipment.
4. New aspects of this contribution include an original interpretation of experimental results obtained from a simple buckling set up.
5. The material can be used directly as a laboratory text.
6. A basic textbook in mechanics of solids is sufficient to accompany the presented materials.
7. The concepts presented have been tested in the classroom. This laboratory experiment was useful in demonstrating end effects and the need for new interpretations.
8. This work emphasizes the need for considering *all* parameters when analyzing actual cases even for very simple set-up.

INTRODUCTION

Understanding the mechanics of the buckling of structural members is an important aspect of civil engineering. Amplification of the deflections and sudden failure caused by axial or in-plane loading must be avoided in any safe design. Therefore, it is important that mechanical and civil engineering students have an opportunity to observe and understand the concepts of buckling and instability through a laboratory experiment.

Because of the size of student classes (commonly 20–25 groups of four to five students) it is impractical to bring columns to failure, due to the prohibitive costs of the materials and machining time involved, as well as safety considerations. Methods based on the extrapolation of elastic behaviour to critical instability are thus needed if one wants to design a practical and repeatable buckling experiment.

This paper describes how the author, working from basic theory, had to modify the initial experimental column buckling set-up in view of the results and difficulties encountered by students. A final laboratory version is presented together with a simple method for an interpretation of the critical buckling load.

PREVIOUS EXPERIMENTS AND OBJECTIVES

The existing buckling experiment carried out by the students used aluminium and stainless steel rectangular strip columns, with both ends resting on guttier-type supports. The determination of the critical load was made according to Southwell's plot [1]. In this method, an initial sinusoidal shape for the column is assumed, using a graphical hyperbolic plot for determining the critical load. For over 5 years this set-up proved to be frustrating for the students and instructors, since the results were not reproducible and showed a poor agreement between the experimental critical load and Euler's theoretical value. The actual type of support was questionable as well as the initial assumptions on the unloaded beam.

The ideas that led to a modified design were as follows.

1. To use different type of supports (fixed supports).
2. To create a known initial deformed shape for the column by applying a perpendicular force at the midspan and then approach the critical load by applying the axial load in steps.

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3. To change the initial deformed shape to confirm the critical load value and check the assumptions made on the material properties and type of end supports.

BASIC BUCKLING COLUMN THEORY

If a vertical elastic column of span L, of minimum moment of inertia I and modulus E, rigidly fixed at the ends is subjected to a horizontal force F at the midspan and an axial force P (see Fig. 1), the differential equation governing the lateral deflection v at the distance x from the top end is

$$EI\frac{d^2v}{dx^2} = -Pv - \frac{Fx}{2} + M_0 \quad (1)$$

where M_0 is the reaction moment at the top. Introducing the dimensionless quantities

$$p = PL^2/4EI$$

$$f = FL^2/EI$$

$$u = v/L$$

$$a = p^{0.5}$$

the deflection at the midspan is given by

$$u_{mid} = \frac{f}{16a^3}\frac{(2 - \sin(a)a - 2\cos(a))}{\sin(a)} \quad (2)$$

and the inverse of u_{mid} can be expanded in series as follows:

$$\frac{1}{u_{mid}} = \frac{16}{f}[12 - \tfrac{6}{5}p - \tfrac{1}{700}p^2 - \tfrac{1}{63\,000}l] \quad (3)$$

This equation shows that the inverse midspan deflection is very close to a linear function of the axial load, since the second degree term is small for the full range of p values (0–10). The buckling load is given by $\sin(p^{0.5}) = 0$ or $p = \pi^2 = 9.87$. The series expansion (two terms) would give $p = 12 \times 5/6 = 10$ which is within 1.3% of the exact answer. Taking three terms will lead to a 0.143% difference.

The conclusion of this first basic case is that by plotting the inverse of the midspan deflection versus the applied axial load, one should be able to determine by extrapolation from the small initial values of p a satisfactory value for the critical load. Repeating the process for different values of the horizontal load F should the produce a set of concurrent lines. When there is no axial load the classical result is obtained:

$$u_{mid} = \frac{f}{192} \quad (4)$$

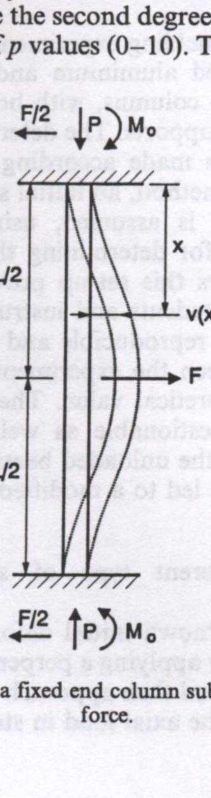

Fig. 1. Buckling of a fixed end column subjected to a midspan force.

FIRST EXPERIMENTAL SET-UP AND PROCEDURES

The initial set-up is shown in Fig. 2 with actual dimensions. The ends of the rectangular cross-sectional beam of aluminium alloy were tightly inserted into solid blocks respectively fixed to the base of the frame and to the load cell-jack unit at the top. The horizontal load was applied by means of a pulley-cable arrangement with dead weights (±0.2 kg taking into account the pulley friction). The axial load was applied with a hydraulic jack and recorded by a load cell (accuracy ±0.5 kg).

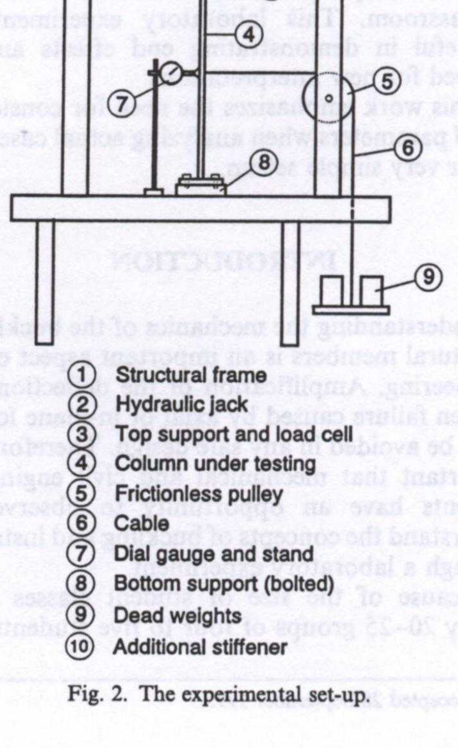

Fig. 2. The experimental set-up.

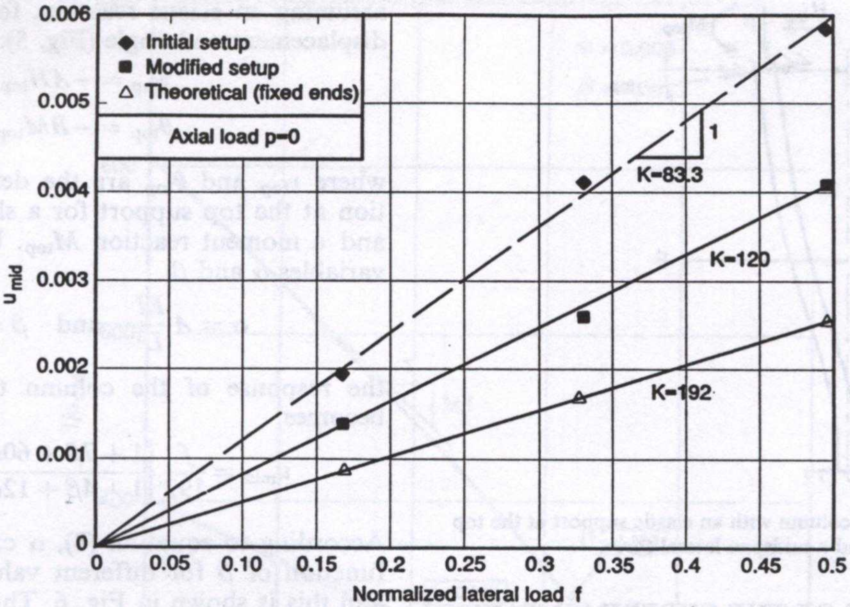

Fig. 3. Normalized force deflection plot for a lateral midspan loading.

The deflections were recorded with a dial displacement gauge (accuracy ±0.01 mm) at the midspan. The length between the support was known to within ±1 mm and the width and thickness of the column to within ±0.05 mm. The corresponding value of I is $8.13 \times 10^{-10}\,m^4$ for a length of $L = 490$ mm.

The initial laboratory procedures given to the 14 groups of students were as follows.

1. Apply a horizontal force of 4 kg and record the displacement at the midspan (v_{mid}). Then increase the axial load by equal increments of 45.4 kg (100 lbs) up to 272.4 kg (600 lbs) recording v_{mid} in each case.
2. Repeat the procedure using $F = 8$ and 12 kg.

After collecting all the students' reports and analysing the results, nine reports were found to be reliable and five had to be rejected. The reasons for rejection were varied, from an obviously poor recording of the dial gauge to misunderstanding the procedures and failure to report the appropriate measurements. However, the nine remaining groups showed consistency in their measurements.

Figures 3 and 4 show the basic plots required, where the results of the nine groups were averaged. The first plot indicates the relation between u_{mid} and f for $p = 0$ (the column without an axial load). Figure 4 is a plot of $1/u_{mid}$ as a function of p for $f = 0.166$, 0.332 and 0.497 ($F = 4$, 8 and 12 kg, respectively).

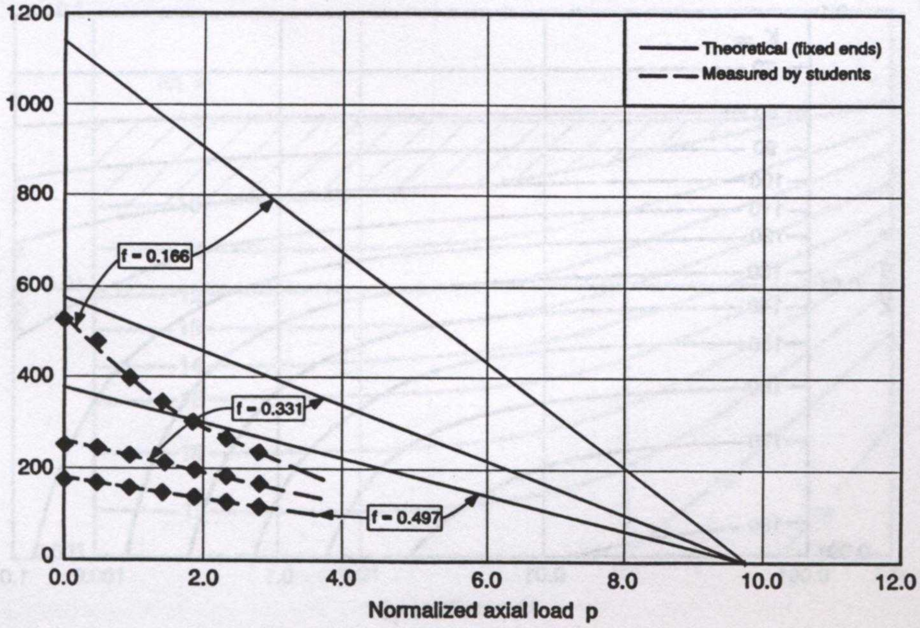

Fig. 4. Normalized (axial force) – (lateral displacement) plot.

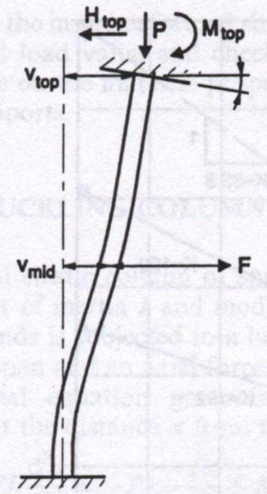

Fig. 5. Buckling of a column with an elastic support at the top end and a midspan lateral force.

DISCUSSION OF THE EXPERIMENTAL RESULTS

The theoretical relation between f and u_{mid} for an aluminium alloy ($E = 70\,\text{GPa}$) and the fixed supports is shown in Fig. 3 and departs obviously from the measured values which can be represented by the line $f = Ku_{\text{mid}}$. This disagreement is reported on the $p = 0$ axis of Fig. 4 which makes the comparison between the theory and the experimental data somewhat discouraging.

In order to interpret and eventually remediate the difference, the discrepancy between the theoretical case and the actual one, the end conditions must be discussed. Since the bottom block was strongly attached to the frame, the top support was questioned and a theoretical analysis of the behaviour of the support has been carried out, assuming an elastic reaction, for both horizontal displacement and angle (Fig. 5):

$$v_{\text{top}} = -AH_{\text{top}} \tag{5}$$

$$\theta_{\text{top}} = -BM_{\text{top}} \tag{6}$$

where v_{top} and θ_{top} are the deflection and rotation at the top support for a shear reaction H_{top} and a moment reaction M_{top}. Using the reduced variables α and β,

$$\alpha = A\frac{EI}{L^3} \quad \text{and} \quad \beta = B\frac{EI}{L} \tag{7}$$

the response of the column to a horizontal f becomes

$$u_{\text{mid}} = \frac{f}{192}\left[\frac{1 + 7\beta + 60\alpha + 96\alpha\beta}{1 + 4\beta + 12\alpha + 12\alpha\beta}\right] \tag{8}$$

According to equation (8), α can be plotted as a function of β for different values of $K = f/u_{\text{mid}}$ and this is shown in Fig. 6. This, however, is not sufficient to select a particular set of values for (α, β) and the plot in Fig. 4 must also be used.

The fundamental equation for elastic beams with an elastic support at the top (α, β) and subjected to a lateral force f and to an axial force p must be solved. However, the solution is not simple and has been worked out using the mathematical software MAPLE [2]. A typical result of this analysis is shown on Fig. 7 where the following can be observed.

1. For p ranging from 0 to 3 the plot is close to a straight line whose slope is inversely proportional to the normalized lateral load f, following the equation

$$\text{slope}\left[\frac{1}{u_{\text{mid}}} \text{ vs } p\right]_{\text{at point } p=2} = \frac{M}{f} \tag{9}$$

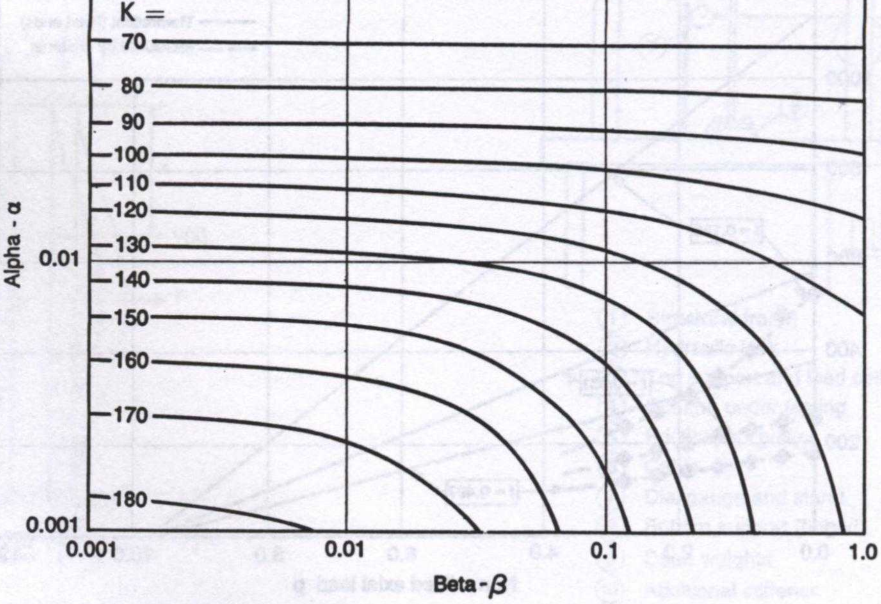

Fig. 6. Relation between the support stiffness parameters and K.

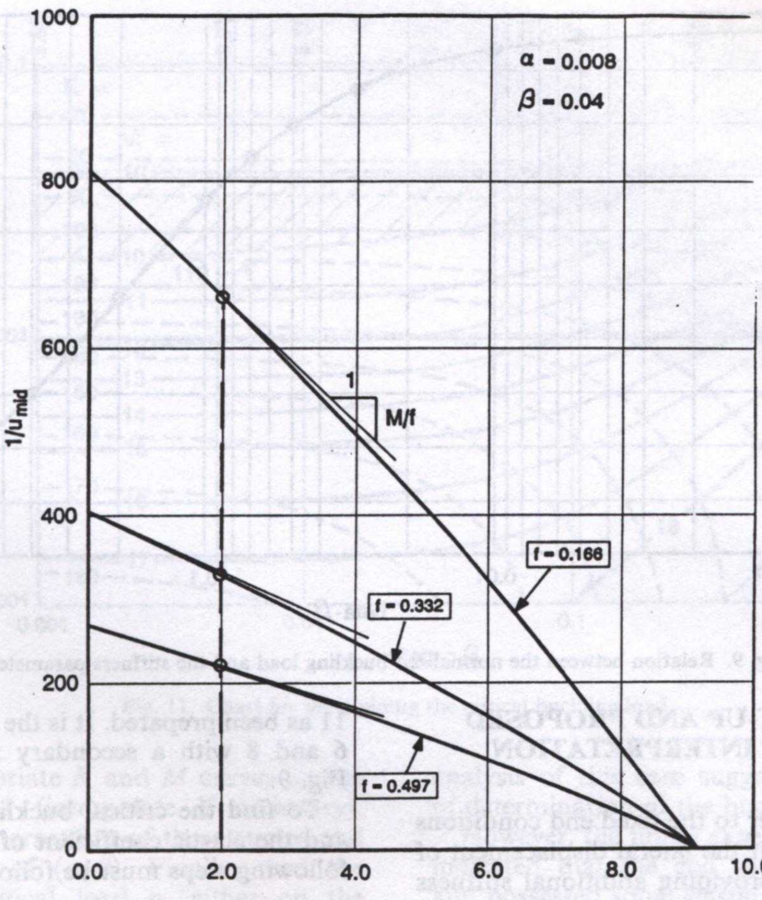

Fig. 7. Buckling plot for a column with an elastic support at the top end.

This parameter M, which is independent of f, can be calculated for different values of (α, β) and the plot shown in Fig. 8 can be obtained.

2. The critical value p_c lies between π^2 (the case of two fixed ends) and $\pi^2/2$ (the case of one fixed end and one pinned end). The value of p_c is not appreciably affected by the value of α and the relation between β and p_c can be plotted as in Fig. 9.

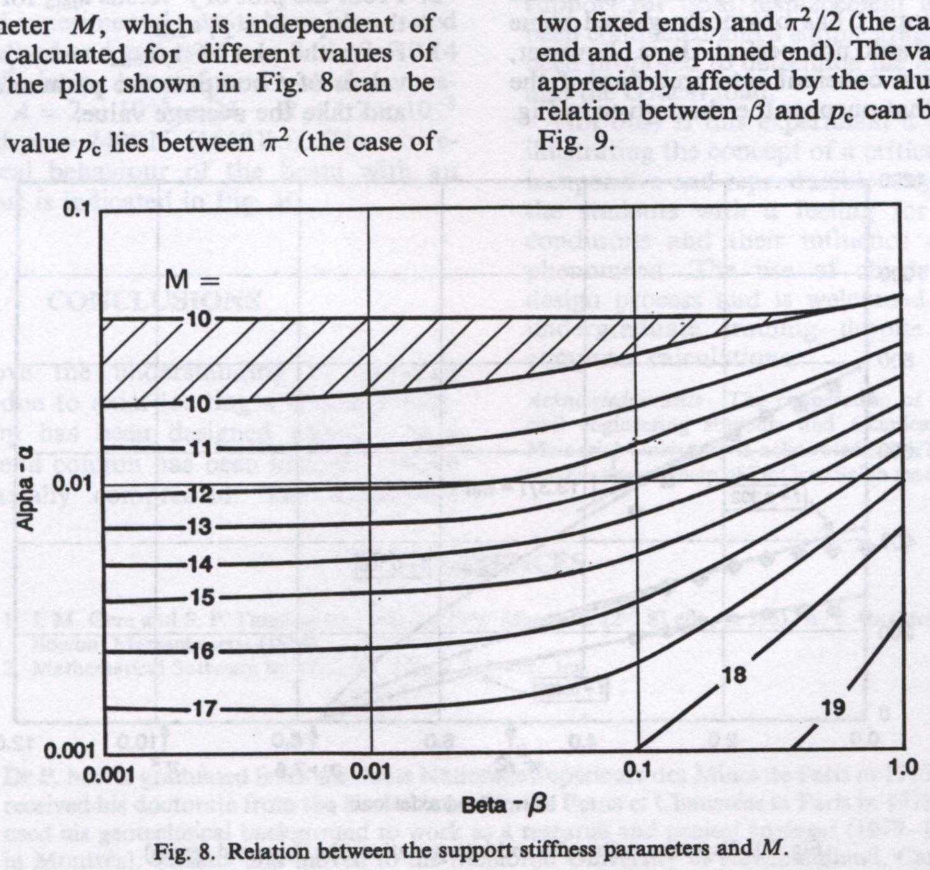

Fig. 8. Relation between the support stiffness parameters and M.

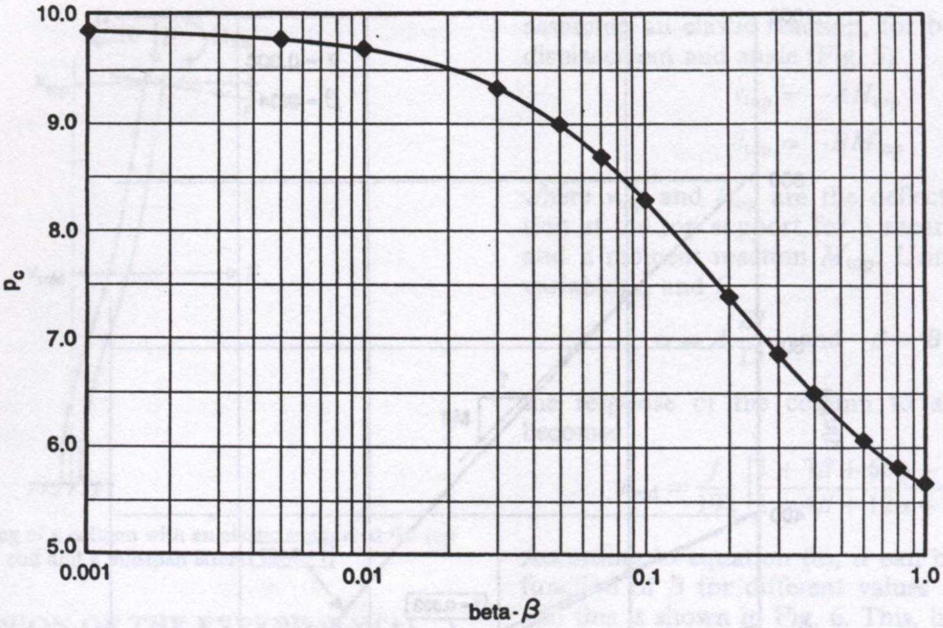

Fig. 9. Relation between the normalized buckling load and the stiffness parameters.

MODIFIED SET-UP AND PROPOSED METHOD OF INTERPRETATION

In order to be closer to the fixed end conditions it was decided to limit the lateral displacement of the top support by providing additional stiffness (see the detail in Fig. 1) at the base of the jack cylinder. The experiment was carried out as before by a team of instructors and the results were analysed by drawing the same plots (Figs 3 and 10). The top support has obviously gained some 'fixity' but it is still not perfect. It is, however, possible to give a coherent interpretation of the difference. For that purpose the chart given in Fig. 11 as been prepared. It is the superposition of Figs 6 and 8 with a secondary x-scale equivalent to Fig. 9.

To find the critical buckling load of the beam and the elastic coefficient of the top support, the following steps must be followed.

1. Normalize all the measured values v_{mid}, F and p to u_{mid}, f and p using the measured L and I and tabulated E.
2. From the plot of f versus u_{mid} for $p = 0$ deduce the slope $K = f/u_{mid}$.
3. From the plot of $1/u_{mid}$ versus p at successive values of f compute the parameter M at $p = 2$ and take the average value.

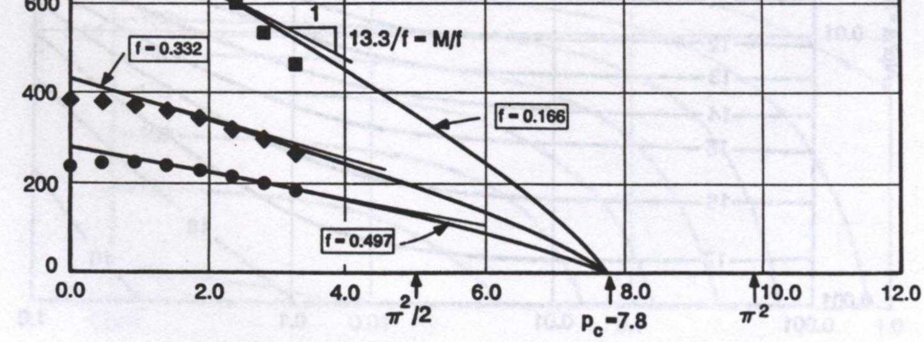

Fig. 10. Experimental results and interpretation (the improved set-up).

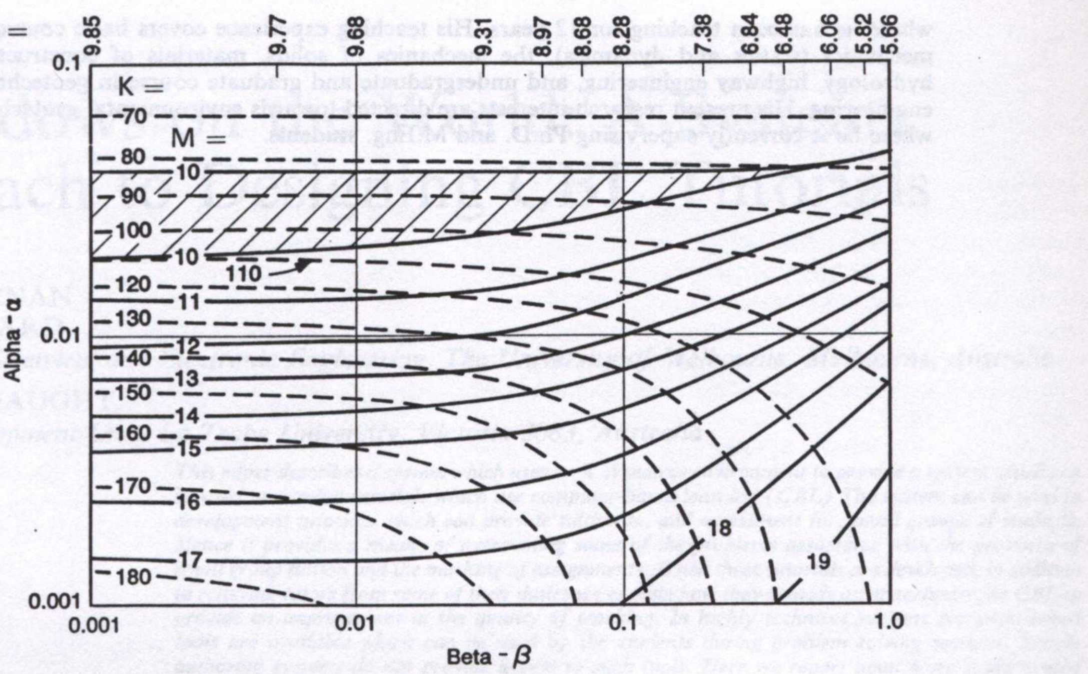

Fig. 11. Chart for determining the critical buckling load.

4. Select the appropriate K and M curves on the chart in Fig. 11 (interpolate if necessary). Determine the intersection of these curves and read the values of (α, β). Finally, read the value of the critical load p_c either on the secondary x-axis of the chart or in Fig. 9.
5. Change the normalized values for actual values using E, I and L.

The modified experimental results have been tested using this method and values of $\alpha = 0.01$, $\beta = 0.14$ and $p_c = 7.8$ have been found which correspond to $A = 2 \times 10^{-5}$ m/N, $B = 1.2 \times 10^{-3}$ $(\text{m N})^{-1}$ and $p_c = 7400$ N (1660 lbs). The corresponding ideal behaviour of the beam with an elastic support is indicated in Fig. 10.

CONCLUSIONS

To improve the understanding of buckling phenomena due to axial loading a simple experimental set-up has been designed where a supposed fixed end column has been laterally loaded and then axially compressed. The theoretical analysis of this case suggested a simple method of determination of the buckling load.

However, the initial experimental results clearly indicated that the end supports were not fixed and possessed some elastic components. This was verified by modifying the initial set-up and increasing its stiffness. A theoretical analysis of the same column having one fixed support and one elastic support for both displacement and rotation has been conducted and a simple graphical method has been proposed to determine the elastic parameters and the critical load.

Not only is this experiment a valuable tool for illustrating the concept of a critical load in a fairly inexpensive and reproducible way, it also provides the students with a feeling for the actual end conditions and their influence on the buckling phenomena. The use of charts is part of the design process and is welcomed at this stage of undergraduate training despite the advent of computer calculations.

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2. Mathematical Software by Waterloo Maple Software Inc.

Dr P. Morin graduated from the Ecole Nationale Supérieure des Mines de Paris in 1975 and received his doctorate from the Ecole Nationale des Ponts et Chaussées in Paris in 1979. He used his geotechnical background to work as a research and project engineer (1979–1982) in Montréal, Canada and moved to the Memorial University of Newfoundland, Canada where he has been teaching for 12 years. His teaching experience covers basic courses in mechanics (statics and dynamics), the mechanics of solids, materials of construction, hydrology, highway engineering, and undergraduate and graduate course in geotechnical engineering. His present research interests are directed towards environmental geotechnics where he is currently supervising Ph.D. and M.Eng. students.