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Programs for Computer-aided Learning in Elastostatics: BEAM–Plane Bending of Beams, and Quadratic Moments of Cross-Sections*

J. MURÍN
M. MÁČAJ
R. BENKOVIČ

Department of Mechanics, Faculty of Electrical Engineering, Slovak Technical University, Ilkovičova 3, Bratislava, Slovak Republic

A program for PC-AT computers has been designed for computer-aided learning of beam plane bending and for evaluation of quadratic moments of cross-sections. The progam is based on the method of initial parameters and on the formulae derived and discussed in detail previously. The description of the tasks to be solved, the input file preparation and evaluation of results are facilitated by computer graphical tools.

INTRODUCTION

THE PROBLEM of plane beam bending is a fundamental part of the mechanics of solids. The task involves the statics of solid bodies/static equations of balance, the method of a supposed section, the law of superposition of effects, the influence of the cross-section upon the mechanical state of the beam/quadratic moments of cross-sections, combined loading of beams, the hypotheses of the strength, deformation, of the beam, etc.

In teaching, analytical methods are generally used to solve these problems since these are essential if the students are to understand the problems fully. Because of time constraints, however, it is often impossible to solve more complicated problems such as complicated loading, static indeterminacy and beam deflection. To a certain degree, this hinders a straightforward demonstration of the topic and, consequently, a better insight into the problem.

Utilizing our experience in teaching, we have decided to develop a program for the PC-AT based on one analytical method to solve the problem of plane bending of one-field beams comprehensively. Computer graphics offer the user an interesting and efficient 'technical game' in which the studied topic is better elucidated and, therefore, fully understood.

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PROGRAM DESCRIPTION

The program BEAM is intended for computer-aided learning of plane beam bending and for evaluation of quadratic moments of the cross-sections. The execution part of the program is written in FORTRAN, the graphical part in C. The program can be implemented on PC-AT computers (at least 286 with/without a coprocessor) with a colour card (at least VGA). The program takes at most 300 K of memory. In the interactive mode, the program can be operated either from the keyboard or by a mouse.

Allowable parameters of the solved problem

Boundary conditions of the beam. The two ends of a one-field beam can be mounted by combining arbitrarily the following bounds:

- fixed or sliding joint;
- free end;
- rigidly fixed end;
- vertically sliding constraint;
- flexible joint (with a given spring constant).

The beam mounting can be either statically determinate or indeterminate.

Beam loading. In the plane of deflection the beam can be loaded by a single load or by a combination of the following types of load:

- vertical concentrated force;
- concentrated moment;

- vertical uniform distributed load;
- vertical triangular distributed load.

The sign of load can be either positive or negative. The number of individual types of loads is limited, for practical reasons, to 10 (which gives a total of 40 different loads).

Cross-section and mechanical properties of the beam. The beam is supposed to have a constant cross-section along its length. Its characteristics (such as cross-sectional area, quadratic moment of the cross-section, reduced cross-section, cross-sectional modules in bending) can be entered individually, or the subroutine PROFILE can be utilized, which calculates the characteristics of the 13 most commonly used beam profiles. A detailed description of the subroutine will be given below.

The type of material is specified by entering its modulus of elasticity in tension (compression).

Input and output data

The input file of the beam can be created manually (by filling in the tables and windows), graphically (by utilizing the facilities of computer graphics) or by combining these two methods. The file must specify the beam dimensions and material, as well as the load and boundary conditions.

The program calculates and displays in a graphical form:

- the solved beam configuration;
- the curves of bending moments and transverse forces;
- the deflection line and the angle of beam rotation;
- normal stress resulting from bending moments, and transverse stress resulting from transverse forces;
- reduced stress, as calculated according to the fifth hypothesis of strength.

Evaluation of quadratic moments of cross-sections

From the basic dimensions of the cross-section that are entered (13 common shapes of cross-sections are available), the subroutine PROFILE calculates the characteristics of the selected profile. This part of the program contains basic relations and formulae for evaluating:

- the cross-section area;
- the coordinates of the centroid;
- quadratic moments of cross-sections with respect to axes passing through the centroid, with respect to arbitrary axes, or, finally, with respect to arbitrarily rotated axes;
- the main and central quadratic moments of cross-sections;
- the radius of inertia.

The calculated results are displayed both graphically and in a table. The axis rotation is controlled by cursor arrows.

METHOD OF SOLUTION

The algorithm utilizes the Euler–Bernoulli equation of beam deflection (see Fig. 1):

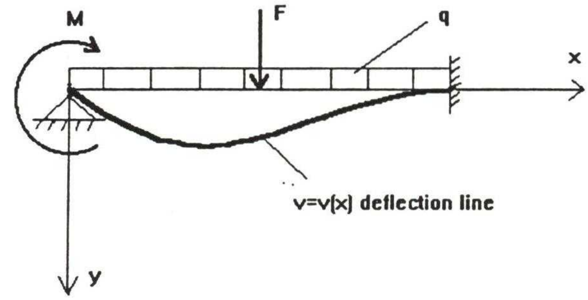

Fig. 1. Plane beam deflection.

$$EI\frac{d^4v}{dx^4}=q \qquad EI\frac{d^3v}{dx^3}=-T$$
$$EI\frac{d^2v}{dx^2}=-M \qquad EI\frac{dv}{dx}=\phi \tag{1}$$

where:

E = modulus of elasticity in tension (compression), I = quadratic moment of the cross-section, $v = v(x)$ describes the deflection line, $T = T(x)$ is the transversal force, $M = M(x)$ is the bending moment and $\phi = \phi(x)$ is the angle of beam rotation.

By solving equations (1) we obtain:

Beam deflection:

$$v(x)=v_0+\phi_0.x-M_0\frac{x^2}{2EI}-T_0\frac{x^3}{6EI}+F_v$$

Angle of beam rotation:

$$\phi(x)=\phi_0+M_0\frac{x}{EI}+T_0\frac{x^2}{2EI}+F_\phi$$

Bending moment:

$$M(x)=M_0+T_0x+F_M$$

Transverse force:

$$T(x)=T_0+F_T$$

The load functions F_v, F_ϕ, F_M and F_T depend on the way in which the beam is loaded. The functions v_0, ϕ_0, T_0 (initical values of deflection, angle of rotation, bending moment and transverse force) depend on the method of beam bounding. For typical files of beam bounding they can be found in [1].

The mentioned formulae are utilized in the executing part of BEAM.

PROGRAM OPERATION AND EXAMPLE

The program is easy to use, which satisfies the requirement that educational software tools should be interactive. It is possible to solve a new task at any time or to solve a previous task with new beam parameters. Wrong input data can be immediately corrected interactively. A hardcopy of the screen can be printed at any time. The user must have a basic knowledge of MS-DOS and of the theory of plane beam bending. The program is provided with an instruction manual [2], where the program operation is described in detail.

Example 1

The curves of bending moments $M(x)$ and transverse forces $T(x)$ are to be found in a beam loaded and mounted as illustrated in Fig. 2. The beam deflection and rotation are also desired. According to the value of the reduced stress, evaluate the loading capacity of the beam if the allowable stress is $\sigma_D = 200$ N/mm^2. Further, the following values are given: $F = 1000$ N, $q = 1000$ N/m, $L = 1$ m, $E = 210\,000$ N/mm^2. The beam has a circular cross-section with diameter $d = 38$ mm.

For the sake of brevity we give only the final windows of the program where the forces, deflections and stresses in the beam are displayed graphically and in a table (the values change with cursor movement). The beam itself is displayed in colours which change with reduced stress (see Figs 3 and 4). It is evident from the results presented that the beam is over-designed.

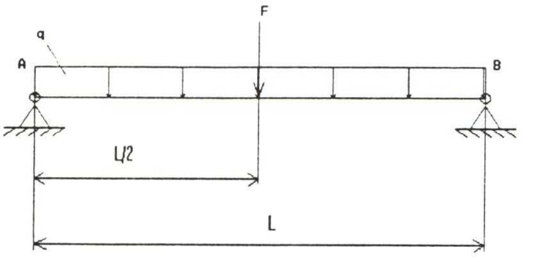

Fig. 2. Example 1.

Example 2

Evaluate the characteristics of the cross-section with respect to axes passing through the centroid, with respect to parallelly shifted axes, and with respect to a rotated axes for a circular cross-section with diameter $d = 38$ mm. Figure 5 presents the final window of the program with the desired quantities.

Example 3

Figures 6 and 7 present the final windows of the program for the additional tasks.

PROGRAM UTILIZATION IN EDUCATION

The programs BEAM and PROFILE are suitable for learning the plane deflection of beams, for evaluation of quadratic cross-section characteristics, as well as for solving many practical problems. The programs can be utilized in giving and evaluating students' homeworks and tests, and/or for elaborating specified projects containing both classical and computer solutions of the task. Graphical presentation of data and results makes the problem easy to understand, and tasks that would be too difficult, laborious and time-consuming for a classical approach can be solved.

Practical training with the programs BEAM and PROFILE can be organized in the following way:

1. The teacher defines and solves, with the students, a selected problem classically (e.g. by

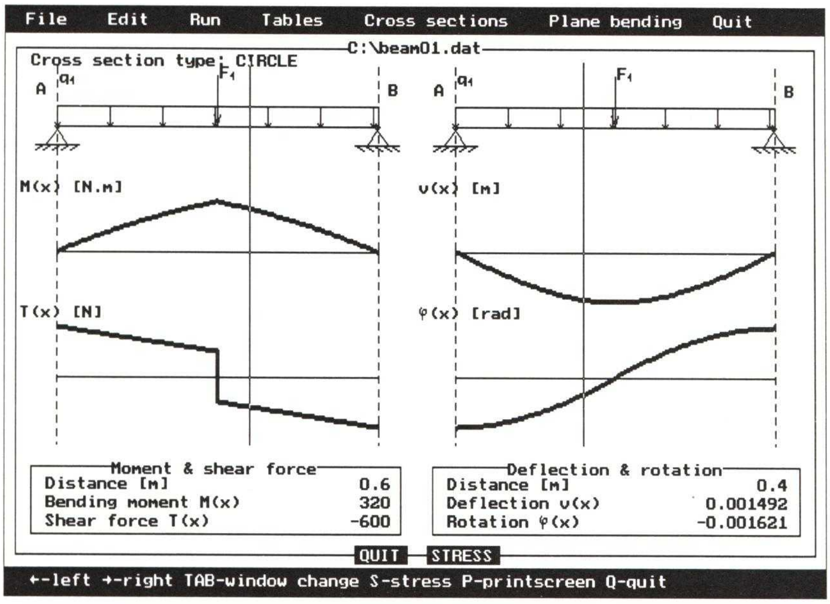

Fig. 3.

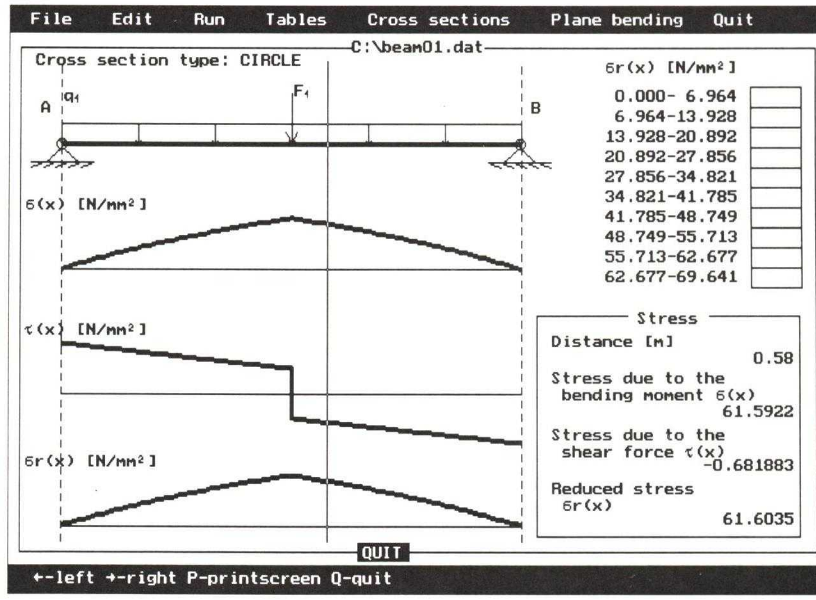

Fig. 4.

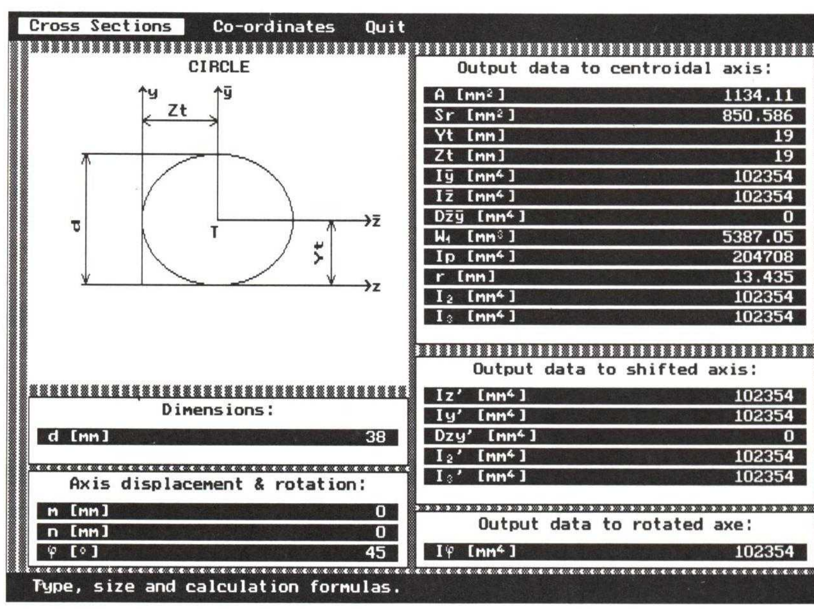

Fig. 5.

using the method of a supposed section and by an approximate differential equation for the beam deflection line).
2. For the same task the input file is created and the beam bending is solved by BEAM.
3. The results of both approaches should be compared and the efficiency and importance of both methods should be discussed (the classical solution is essential for understanding the physical background of the solution properly).
4. After changing the input parameters a set of tasks can be solved and the influence of individual parameters on the results can be observed. The program PROFILE can be used to compute the quadratic moments of the selected beam cross-section.
5. Students can then work on their own designs.

Naturally, the scope of these programs depends on the user's creativity and on applications for which they are used.

BEAM is available on request from the first author.

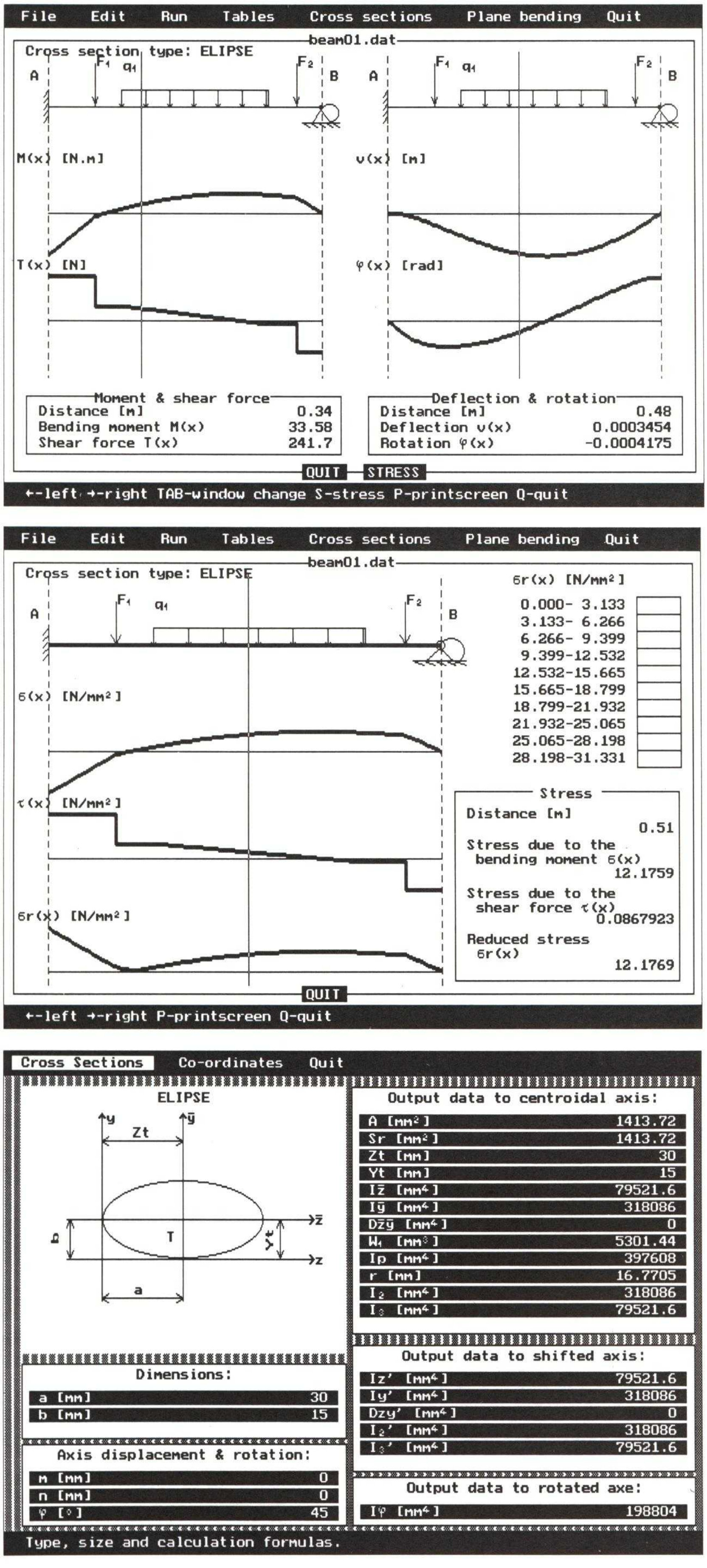
File Edit Run Tables Cross sections Plane bending Quit
beam01.dat
Cross section type: ELIPSE
A
B
M(x) [N.m]
T(x) [N]
v(x) [m]
φ(x) [rad]
Moment & shear force
Distance [m] 0.34
Bending moment M(x) 33.58
Shear force T(x) 241.7
Deflection & rotation
Distance [m] 0.48
Deflection v(x) 0.0003454
Rotation φ(x) -0.0004175
QUIT
STRESS
←-left →-right TAB-window change S-stress P-printscreen Q-quit
File Edit Run Tables Cross sections Plane bending Quit
beam01.dat
Cross section type: ELIPSE
σ(x) [N/mm²]
τ(x) [N/mm²]
σr(x) [N/mm²]
0.000- 3.133
3.133- 6.266
6.266- 9.399
9.399-12.532
12.532-15.665
15.665-18.799
18.799-21.932
21.932-25.065
25.065-28.198
28.198-31.331
Stress
Distance [m]
0.51
Stress due to the bending moment σ(x)
12.1759
Stress due to the shear force τ(x)
0.0867923
Reduced stress σr(x)
12.1769
QUIT
←-left →-right P-printscreen Q-quit
Cross Sections Co-ordinates Quit
ELIPSE
Zt
T
a
b
Yt
Output data to centroidal axis:
A [mm²] 1413.72
Sr [mm²] 1413.72
Zt [mm] 30
Yt [mm] 15
Iz̄ [mm⁴] 79521.6
Iȳ [mm⁴] 318086
Dz̄ȳ [mm⁴] 0
W₁ [mm³] 5301.44
Ip [mm⁴] 397608
r [mm] 16.7705
I₂ [mm⁴] 318086
I₃ [mm⁴] 79521.6
Dimensions:
a [mm] 30
b [mm] 15
Axis displacement & rotation:
m [mm] 0
n [mm] 0
φ [°] 45
Output data to shifted axis:
Iz' [mm⁴] 79521.6
Iy' [mm⁴] 318086
Dzy' [mm⁴] 0
I₂' [mm⁴] 318086
I₃' [mm⁴] 79521.6
Output data to rotated axe:
Iφ [mm⁴] 198804
Type, size and calculation formulas.

Fig. 6.

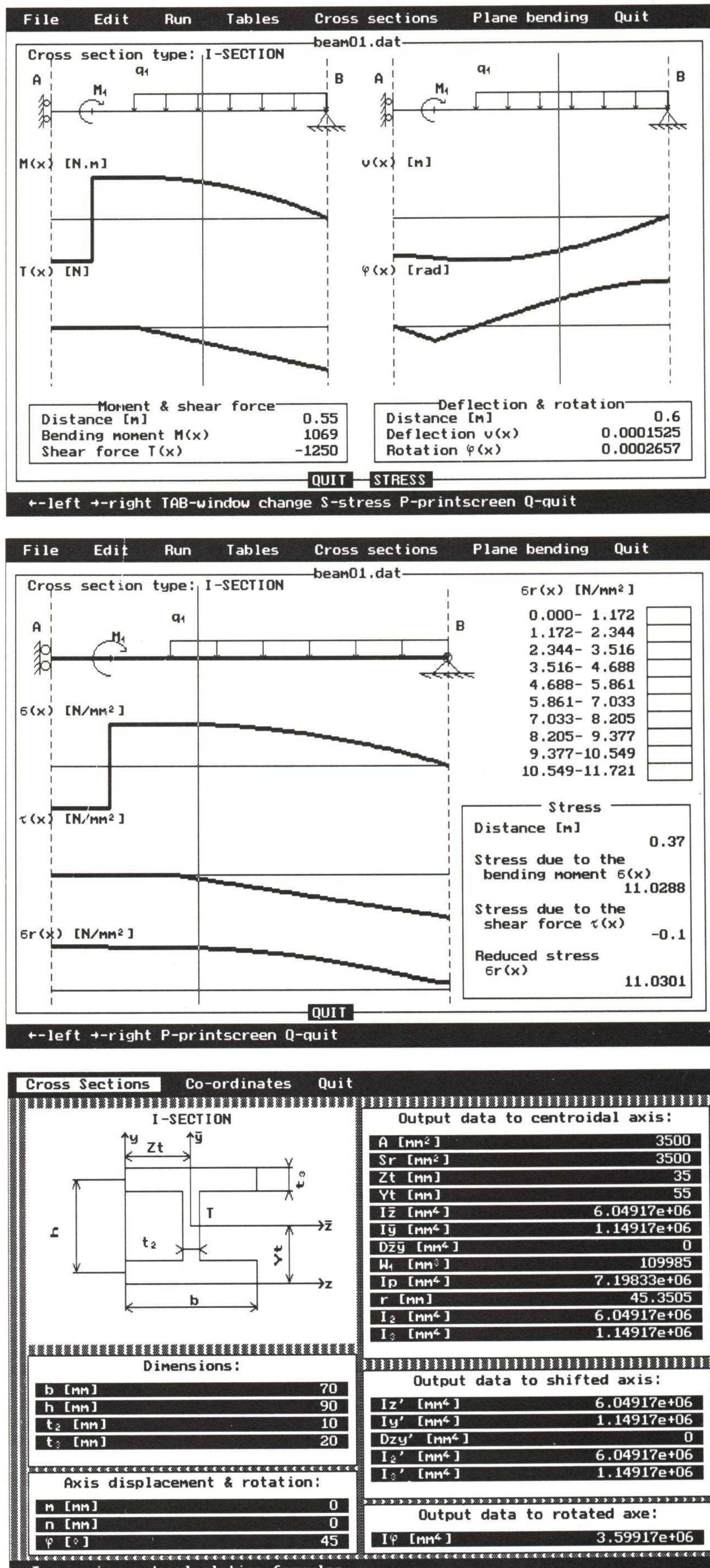
File Edit Run Tables Cross sections Plane bending Quit
beam01.dat
Cross section type: I-SECTION
A
B
q1
M1
M(x) [N.m]
T(x) [N]
v(x) [m]
φ(x) [rad]
Moment & shear force
Distance [m] 0.55
Bending moment M(x) 1069
Shear force T(x) -1250
Deflection & rotation
Distance [m] 0.6
Deflection v(x) 0.0001525
Rotation φ(x) 0.0002657
QUIT
STRESS
←-left →-right TAB-window change S-stress P-printscreen Q-quit
File Edit Run Tables Cross sections Plane bending Quit
beam01.dat
Cross section type: I-SECTION
σ(x) [N/mm²]
τ(x) [N/mm²]
σr(x) [N/mm²]
σr(x) [N/mm²]
0.000- 1.172
1.172- 2.344
2.344- 3.516
3.516- 4.688
4.688- 5.861
5.861- 7.033
7.033- 8.205
8.205- 9.377
9.377-10.549
10.549-11.721
Stress
Distance [m] 0.37
Stress due to the bending moment σ(x) 11.0288
Stress due to the shear force τ(x) -0.1
Reduced stress σr(x) 11.0301
QUIT
←-left →-right P-printscreen Q-quit
Cross Sections Co-ordinates Quit
I-SECTION
Output data to centroidal axis:
A [mm²] 3500
Sr [mm²] 3500
Zt [mm] 35
Yt [mm] 55
Iz̄ [mm⁴] 6.04917e+06
Iȳ [mm⁴] 1.14917e+06
Dz̄ȳ [mm⁴] 0
W1 [mm³] 109985
Ip [mm⁴] 7.19833e+06
r [mm] 45.3505
I2 [mm⁴] 6.04917e+06
I3 [mm⁴] 1.14917e+06
Dimensions:
b [mm] 70
h [mm] 90
t2 [mm] 10
t3 [mm] 20
Output data to shifted axis:
Iz' [mm⁴] 6.04917e+06
Iy' [mm⁴] 1.14917e+06
Dzy' [mm⁴] 0
I2' [mm⁴] 6.04917e+06
I3' [mm⁴] 1.14917e+06
Axis displacement & rotation:
m [mm] 0
n [mm] 0
φ [°] 45
Output data to rotated axe:
Iφ [mm⁴] 3.59917e+06
Type, size and calculation formulas.

Fig. 7.

REFERENCES

1. P. Pilkey and Yu Chang, *Modern Formulas for Statics and Dynamics*. McGraw-Hill, New York (1982).
2. J. Murín and M. Máčaj, *Program and Users' Manual*. Project Commenius. Bratislava (1992).