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Finite Element Methods in Mechanical Engineering Education*

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The paper describes the author's experience of teaching finite element methods to mechanical engineering undergraduates, over a period exceeding 20 years. The paper also describes the author's more recent experiences of teaching the use of finite element techniques for design purposes where computer packages such as PIGS and PAFEC are adopted. Examples of students' coursework are given, together with references of published works.

1. INTRODUCTION

THE importance of finite element methods in mechanical, civil and aeronautical engineering, has grown rapidly in recent years, in parallel with the major advances made with computational hardware and software. This double-fronted advance was necessary, because without digital computers the finite element method would be virtually useless. The popularity of the finite element method is because it is one of the most powerful methods for solving partial differential equations that apply over complex shapes, with complex boundary conditions and loads.

The method owes its infancy to the matrix displacement method, which first appeared in the 1940s [1, 2], when it was used to increase the structural efficiency of aircraft. In those days, computational power was relatively poor; additionally, computers were unreliable and very expensive, and beyond the reach of most companies. In fact, in order to implement the matrix displacement method, teams of operators of electro-mechanical calculators, often took several weeks to invert a matrix of modest size.

The true finite element method was invented by Turner *et al.* [3], when Professor Ray Clough of Berkeley, California, developed in-plane elements for his PhD thesis. A similar method to the finite element method was invented even earlier, by Courant [4]; even before the invention of the digital computer!

Since then, considerable developments have taken place in the field of finite elements [5] and these developments have been aided by Intel's invention of the microprocessor in 1970. The development of microprocessors, integrated circuits and associated electronic and electro-mechanical components, have made computers much more powerful than envisaged in the 1950s, and additionally, computers are now within the purchasing power of most companies.

The present author realised the potential of the finite element method in the 1960s and persuaded the then Head of Department at Portsmouth, to allow its introduction into undergraduate courses in the early 1970s. In those days it was fortunate that hand-held electronic calculators had appeared in high street shops, as these were a necessary prerequisite for implementing the finite element method at undergraduate level. The use of these hand-held machines, enabled undergraduates to analyse plane and space pin-jointed trusses, and also, continuous beams and rigid-jointed plane frames. The true finite element method was also introduced at undergraduate level where it was used to develop stiffness and mass matrices for rods, beams, and in-plane plate elements. Throughout the decade the course was developed so that it also included the buckling and vibration of structures. In those days, student numbers were small and the finite element option rarely attracted more than a dozen students.

Throughout the 1980s, developments in the course took place, and in addition to the undergraduates requiring a theoretical knowledge of finite elements, they were required to design structures with the aid of microcomputers such as the Commodore PET and the Sinclair QL. At this stage, the present author had written interactive user-friendly software [6] which the students used with little instruction. The method became established as valid and was used extensively in industry; as a result of this, student numbers grew in this option. Additionally, new topics such as computer aided engineering (CAE) started to appear in the undergraduate courses. These computer-aided engineering courses, usually required the use of finite elements.

Today, the finite element method can be applied to many different branches of engineering science, including structural analysis, vibration, network problems, heat transfer, fluid flow problems, seepage through porous media, acoustics, electrostatics, magnetostatics and many more fields.

Software houses, supporting finite element

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methods, have mushroomed and many major packages are available on both microcomputers and mainframes. Indeed, Ross [7] predicted that this would happen and it is likely that this process will continue to grow in the foreseeable future, so that these packages will be available on palm-top computers. Indeed, the main limitations on these machines will be the sizes of their keyboards, screens and floppy disk ports. The size of the hard disk is less likely to be a problem, as the current electro-mechanical hard disk will probably be replaced by some type of electronic equivalent. When such machines and software are within the purchasing power of the average student, it is likely that the teaching and assessment of undergraduate courses will be revolutionized.

At Portsmouth, the theory and application of finite element methods is offered as a final year option. Additionally, through the use of PIGS and PAFEC, the application of finite element techniques is offered as part of a CAE final year option and as part of a compulsory CAE course in the second year of the BEng and the HND courses. At Portsmouth, finite element methods are considered to be so important for mechanical engineers that they form part of a compulsory CAE course for second year students, so that these students will have some exposure to this topic. A brief description of each of these courses will now be given.

2. FINITE ELEMENT COURSES

2.1 *Final year finite element option*

This course is largely of a theoretical nature, where the students learn the fundamental principles of the matrix displacement method and the finite element method. They are taught to apply the matrix displacement method to plane and space pin-jointed trusses, and to continuous beams and rigid-jointed plane frames. Additionally, they are taught finite element theory and, with the aid of the principle of virtual work and the method of minimum potential, they are taught how to develop stiffness and mass matrices for some finite elements. The methods are extended to the buckling and vibration of structures and to the non-linear vibrations of structures. The course consists of thirty-three 'lectures', each of 1.5 hours duration, plus 3 laboratory periods, each of 2 hours duration.

As far as the so-called 'lecture' period is concerned, the present author artificially divides this approximately into a half hour lecture, followed by a 1 hour tutorial. He finds this a satisfactory combination, as students have great difficulty in fully concentrating for a formal lecture which exceeds half an hour. Also, students prefer tutorials to formal lectures for mathematical type subjects. This arrangement is aided by a book written by Ross [8], which has been written specifically for this course. Students are required to have this book, because its use enables the lecture/tutorial time to be divided into the ratio 1:2, and additionally, its use enables the course to be completed by the end of the second term, so that the bulk of the third term can be used for revision.

The half hour lecture involves the use of transparencies for an overhead projector, where the relevant pages of the book are projected on to a white screen. After this short lecture, the students are advised to read again the appropriate pages of the book prior to tackling the tutorial examples. The students are reassured that, as this material is new to them, they cannot expect to understand it immediately, and that understanding can best be achieved by reading the book and by working though the examples. The present author has worked solutions for all the tutorial examples in his book, and because tutorial classes may exceed 50 students, spare copies of the worked solutions are kept, which students are allowed to consult from time to time. Thus, the more able students are not delayed as, usually, their errors can be found by consulting the official solution. Additionally, more time can be devoted to the slower student, who in any case can consult an official solution which other students have completed.

For examples which worry a large number of students, the author will solve these problems on the blackboard, using 'chalk and talk'. Thus, it can be summarised that the half hour formal lecture is delivered with the aid of an overhead projector and a purpose written text book, and the tutorial is delivered, sometimes, with the aid of 'chalk and talk'. The present author considers the latter method to be unsuitable for lectures, as the reproduction of matrices of large order, on the blackboard, which are then reproduced in the students' notes, is too time consuming and prone to error. The 'chalk and talk' method that is used in tutorials is preferable to the use of an overhead projector, as problem solving can be more definitive and deliberate, and in any case, more interactive. Additionally, if students require further explanation of parts of the course that they are unsure about, regardless of which term they require this information, this is no problem, as the students 'notes', namely Ross's book is virtually error free, and the 'notes' are the same as the lecturers' 'notes' and the same as those 'notes' being projected onto the screen.

It is considered by the present author, that the process described is a powerful and valuable method of student-centred learning. The students are assessed in finite element theory when they take a formal 3 hour examination at the end of the third term. The examination consists of a paper containing 6 questions, where the students are expected to attempt a maximum of 4 questions. This formal examination is worth 85% of the subject mark, the remaining 15% coming from a continuously assessed computer-aided assignment.

As some of these students are not very familiar with commercial computer packages, and also as the present author does not wish to overload the mainframe with students' computer assignments, he requires the CAE for this option to be carried out on microcomputers, with the aid of his software [6].

Each student is presented with the following guidance and requirements for his/her assignment. The students are required to apply an in-plane plate element to a practical problem.

Stress analysis assignment. Using the supplied program 'PLANESTRESS', OR 'STRESS 3F.EXE', design and analyse a suitable structure undergoing in-plane forces.

The in-plane problem can take any form, providing you use between 20 and 100 elements.

Typical structural investigations could be:

(1) transverse strength of a super-tanker;
(2) design of an earth dam,
(3) plate with a crack in it,
(4) gear tooth.

Loads you may wish to consider would be:

(1) wind and snow,
(2) hydrostatic loads,
(3) self weight,
(4) any other in-plane loads.

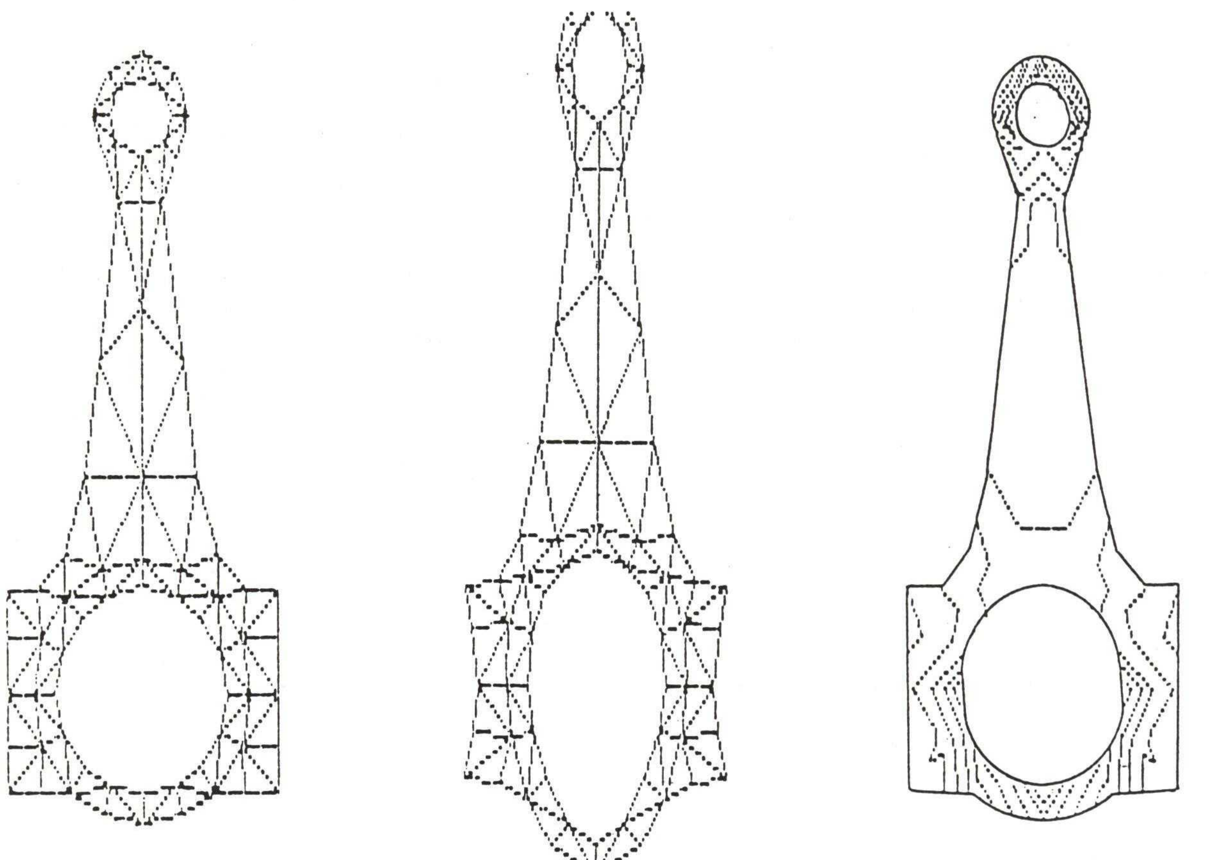
Your report should include:

(a) summary,
(b) introduction to FEM, etc.,
(c) the task, and description of program capabilities,
(d) your chosen design, with load cases,
(e) mathematical model/s,
(f) results and their interpretation,
(g) discussion and conclusions,
(h) references.

N.B. Plane stress problems are for *thin* plates and *plane strain* problems are for *thick* plates.

From the above information, it can be seen that this stress analysis assignment is quite open-ended, and as a result of this, students with a vivid imagination and/or with previous industrial experience, thoroughly enjoy the assignment. Some students have neither much imagination nor industrial experience, and these students are advised to consult books on structures, in the library. In all cases, students are advised to discuss their proposed assignment with either the author or his assistant prior to preparing his/her mathematical model.

In all the years that the present author has been associated with this work, he has not found any evidence of plagiarising. Screen dumps on a dot matrix printer, of a typical assignment are shown in Fig. 1.



CON-ROD DEFLECTED FRAME SHEAR STRESS PLOT

Fig. 1

2.2 *Compulsory second year computer-aided engineering*

For this course, the students are initially presented with an overview of the finite element method, together with its merits and limitations. Additionally, the students are provided with notes on PIGS and PAFEC and are required to consult Ross's books [8, 9] on the finite element method. These books have been written especially for teaching purposes, by an author who has over 20 years of experience of teaching this subject at undergraduate level. The finite element section of computer-aided engineering represents about one-third of this course, and it is continuously assessed. The students are divided into groups of about 12, and each group meets for a total of 4 times, for a period of 1.5 hours duration for each meeting.

In their very first meeting, the students are required to tackle an in-plane plate problem, which takes the form of an end-loaded cantilever, represented by 2 8-node isoparametric elements [10]. Students are first taught how to use PIGS as a pre-processor, when they create the geometry and topology of their mathematical models. Additionally, PIGS will be used to restrain and load the model and then through the module's option, the students will enter the material properties of their model, together with the plate thickness and other details.

When this is complete and prior to leaving PIGS the students will create a .DAT file which is required for analysing the structure via PAFEC. After analysing the structure via PAFEC, the students will retrieve their .BS files, created by PAFEC, for post-processing on PIGS. The post-processor enables the students to obtain the deflected form of the structure, together with various stress contours.

About half the class will complete this exercise by the end of the first period. When the class meets for the second time, those students who have successfully carried out the first exercise are sent to the photoelastic laboratory, where they are scheduled to carry out a photoelastic experiment on an araldite plate with a hole in it. In the mean time, the remaining students continue to complete their first exercise. About half-way through the second period, the sub-group who have not carried out the photoelastic experiment are sent into the photoelastic laboratory where they, in turn, carry out the photoelastic experiment. To achieve this arrangement, the author employs an assistant.

The plate with a hole in it, is shown schematically in Fig. 2, where it can be seen that the plate has 2 axes of symmetry.

The students are required to analyse the plate by 3 methods, as follows:

(a) by a photoelastic experiment,
(b) by the use of a stress concentration formula [11],
(c) by the finite element method.

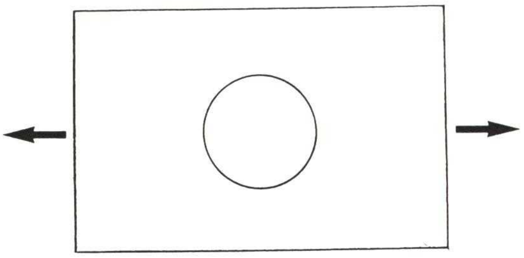


Fig. 2. Holed plate under tension.

Each student is given quite precise details of what is required in his/her report, and he/she is also given the deadline date by which the report is required. As the plate has 2 axes of symmetry, only one-quarter of the plate needs to be analysed, as shown in Fig. 3. To avoid plagiarising, each student is given a different value for the dimension 'L', as shown in Fig. 3. The students are then instructed how to divide their model into PAFBlocks or super-elements, as shown in Fig. 4; this process further decreases the possibility of plagiarising.

Prior to entering details of their PAFBlocks into the computers, the students are required to calculate the coordinates of the nodes of their models, which should be approved by the author or his assistant. The students are required to make the angle θ, in Fig. 4, between 30 and 60°. They are repeatedly warned not to use badly shaped elements.

After the PAFBlocks have been created, the students are given individual instruction on how to mesh their PAFBlocks, so that they can create a model with between 70 and 140 8-node isoparametric elements. The teaching of this topic is very labour intensive, as the students have to be guided throughout the exercise.

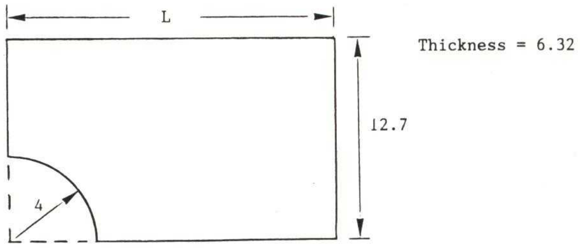


Fig. 3. Quadrant of plate (dimensions in mm).

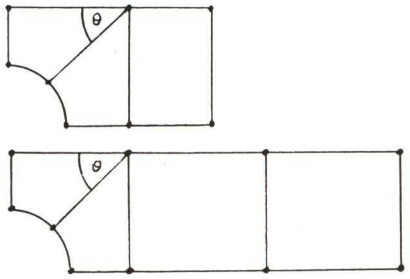


Fig. 4. Models divided into PAFBlocks or super-elements.

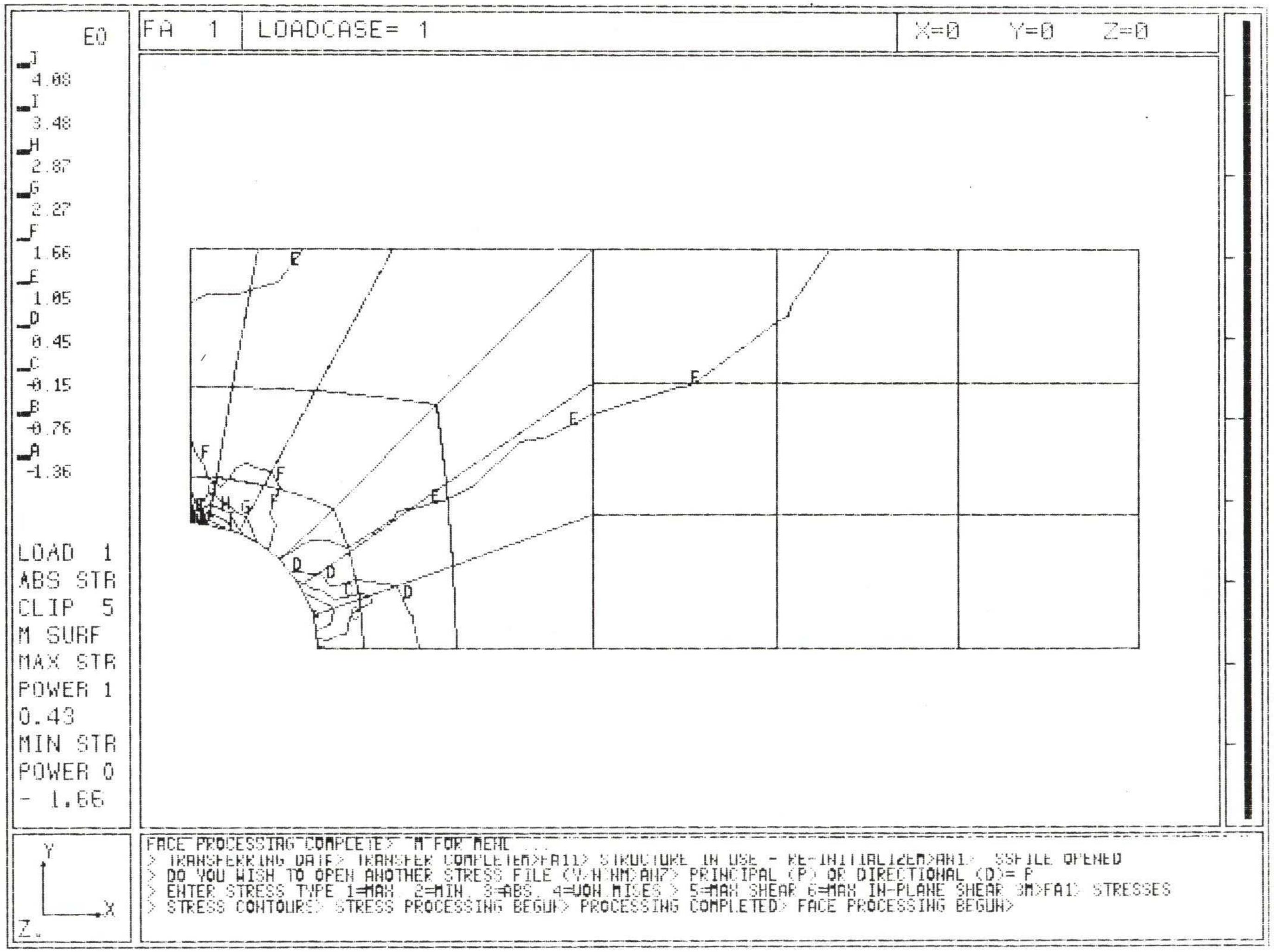


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Fig. 5. Absolute principal stress contours.

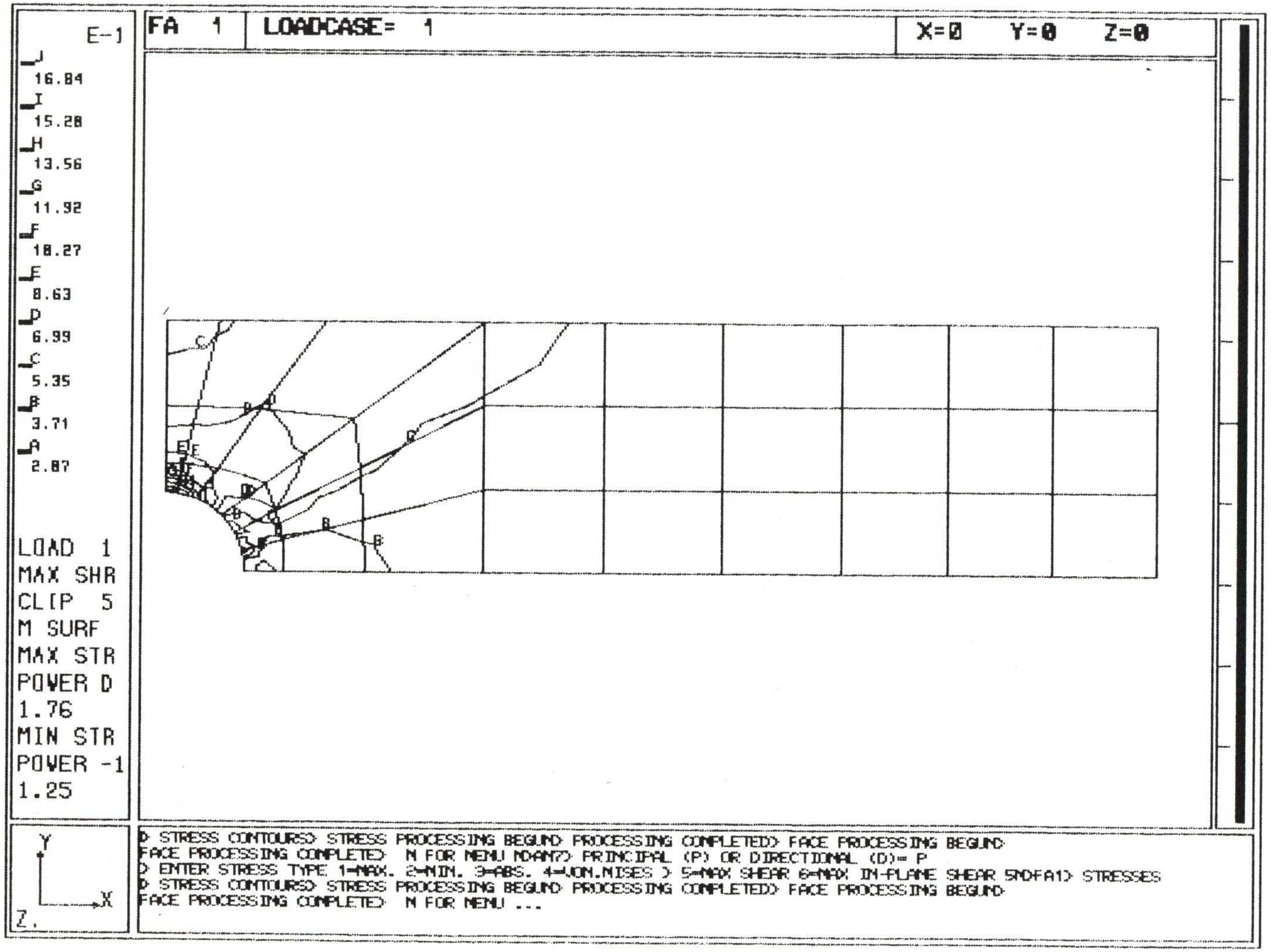

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Fig. 6. Maximum shear stress contours.

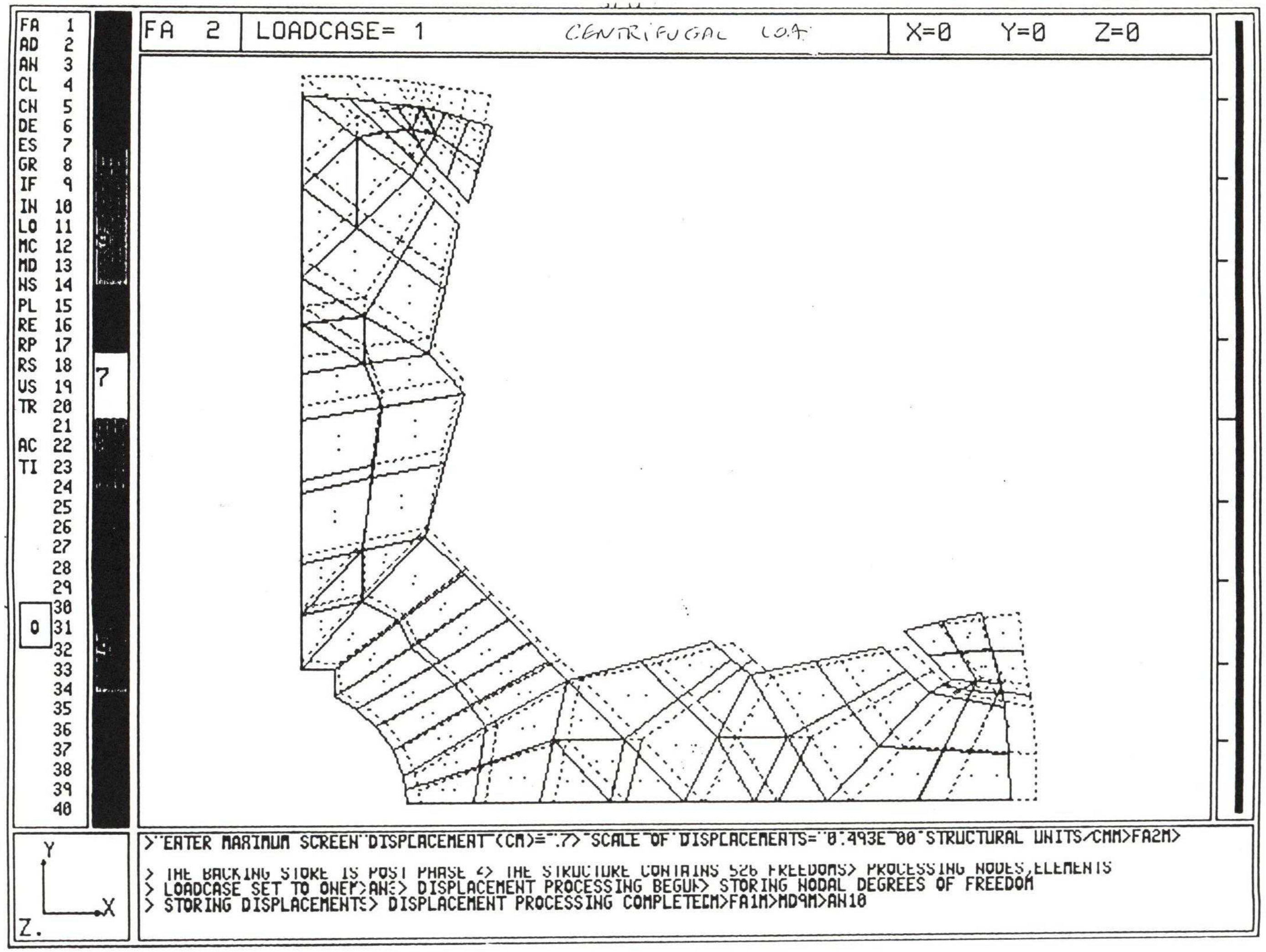

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Fig. 7. Deflected form of clutch plate.

Some students complete their laboratory based work by the end of their third period, and the bulk of the students complete their laboratory work by the fourth and last meeting. The remaining students are required to book a terminal during their private time, and if required, they are given individual instruction by the author's assistant. In many cases, the students are helped by their peers.

It is very important for the lecturer and his assistant to be thoroughly familiar with finite element methods, stress analysis and the various software packages, as students can run into all sorts of problems, which require immediate expert trouble-shooting assistance. Examples of students' work are shown in Figs 5 and 6.

2.3 *Final year computer-aided engineering option*

In this case, the students have alrady had exposure to PIGS and PAFEC, as they have met these packages in their second year. The assignment here is integrated, where the students have previously designed an artefact with the aid of the geometric modeller BOXER and the drawing package DOGS. The finite element section of this course consists of 5 periods, each of 2 hours duration, and the students are expected to analyse 1 component of their artefact. The artefact used last year was a centrifugal clutch, which each student designed individually using BOXER and DOGS. Each student's design was different from those of his/her peers, and each student was required to design 1 component of his/her clutch via PIGS and PAFEC.

The component was selected while using BOXER, and transferred from BOXER to DOGS, and then to PIGS and PAFEC, where it was eventually analysed in a manner similar to that described in Section 2.2. It must be emphasised to the reader that a design made in dimensions of millimetres can lead to serious problems in PAFEC if a dynamic analysis is required, since in a dynamic analysis, it is inadvisable to mix millimetres with Newtons and kilograms.

These assignments were quite successful, but many students required additional time and further personal instruction to achieve their goals. An example of students' work is shown in Fig. 7.

2.4 *Second year HND computer-aided engineering option*

This course is integrated in a similar manner to that described in Section 2.3, except that the artefact to be designed is much simpler. In this case, the finite element option of CAE consists of 5 meetings, each of 2 hours duration. Here again, the students often require additional time and further personal instruction.

3. CONCLUSIONS

Most of the students thoroughly enjoy the final year finite element option. They like the method of delivering the course and its definitiveness, because a purpose-written text book is used. Motivation is high and there are very few failures. This course appeals to students of all calibres, and especially to high flyers. The author seldom experiences discontent amongst the students and there is usually a happy atmosphere in the classroom. Additionally, the students realise that the finite element method is, perhaps, the most important topic in mechanical engineering science. Indeed, the success of this course has spread far and wide, and many major industrialists take pleasure in recruiting employees from this course.

The CAE courses are very important, and students find this topic difficult to grasp, additionally the lecturer finds the topic difficult to deliver. The present author has taught 9 different subjects during his 28 years as an academic, but he finds that the teaching of CAE is the most strenuous. Plagiarising is practically non-existent, but may rear its ugly head if student:staff ratios become too large. Progress, however, is being made with the teaching of CAE, as computer hardware and software continue to be improved, and as more experience is gained in teaching the subject. It is expected that, in the future, CAE may also include finite difference and boundary element methods.

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He left Vickers in 1964 and joined Constantine College of Technology (now Teesside University) as a Lecturer in Civil and Structural Engineering and in 1966, he took up his present post.

Dr Ross is the author of over 70 papers and 10 books, published by international publishers, and for this work, he was awarded a DSc in 1992, by the Council for National Academic Awards.