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Transient Transmission Line Analysis Using Lattice Diagrams*

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This paper presents the transient behavior of a lossless transmission line by means of lattice diagrams. Resistive, capacitive and inductive loads were considered and implemented in the C programming language to be run on IBM PS/2 computers. The developed program has been tested and subsequently used in undergraduate electromagnetic courses. The theory described here can also be used to enhance understanding of the noise performance and the ringing problems related to the multiple reflections associated with microprocessor bus systems

INTRODUCTION

IN SOME electrical engineering courses, it is important for students to understand the phenomena that affect the performance of low power electrical systems, e.g. ringing problems in microprocessor bus systems and the idea of using passive and active terminations [1, 2]. The effect of impedance mismatch repreented in interfacing memory, port or any other systems peripheral to the microprocessor can be understood through the transient analysis of transmission lines. Noise in digital systems due to multiple reflections and the effects of conductor size in setting the characteristic impedance are important factors determining the noise performance of printed circuits. This can also be explained by utilizing transmission line transient analysis. In electromagnetics courses where the subject of transmission lines is covered, the transient and steady-state analyses are important considerations. Students in mechanical engineering can benefit from this work since the analysis presented here can be applied to the transient performance of fluid mechanics, which can be treated in a similar way to transmission lines.

Currently, the material discussed here is covered in our electrical engineering curriculum through an undergraduate electromagnetic course. The course title is 'Electric and Magnetic Fields' and is a 300 junior-level course. In the Purdue University system, this course is EE311 and is required for all electrical engineering tracks. Two prerequisite courses are required: one physics course covering electricity and magnetism (in the Purdue system this is Phys251) and another mathematics course where linear vector algebra is required (Maths262 in our system). Static electric and magnetic fields are covered in the first half of the semester. This is followed by Maxwell's equations, plane waves and transmission lines in the second half of the semester. The distribution of the course material over one semester period is as follows:

1. Coulomb's law and the electric field intensity; electric flux density (Gauss's law); energy and potential; conductors, dielectrics and capacitance; Poisson's and Laplace's equations (4 weeks).
2. The steady magnetic fields (Biot–Savart; Ampère's circuital laws; magnetic flux and flux density; scalar and vector magnetic potentials); magnetic forces; materials; and inductances (4 weeks).
3. Time-varying fields and Maxwell's equations (Faraday's law; displacement current; Maxwell's equations; differential and integral forms; retarded potentials) (1 week).
4. The uniform plane wave (wave motion in free space and perfect dielectrics; plane waves in lossy dielectrics, the Poynting vector and power considerations; propagation in good conductors: skin effect, reflection of uniform plane waves, standing wave ratio) (3 weeks).
5. Transmission lines (transmission line equations; parameters; graphical methods; transient of transmission lines) (3 weeks).

The discussion of the fifth topic is classified into two categories:

1. Single frequency operation (sine wave function) where the analytical and graphical methods can be used.
2. Multi-frequency operations.

The second category is used in some practical applications such as computer networks where pulsed signals may be sent through the line. From

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Fourier analysis, a pulse can be treated as a superposition of waves of many frequencies (fundamental and harmonics). Thus sending a pulsed signal on the line is the same as sending simultaneous different frequencies on the line. As in circuit analysis, when a pulse generator connected to a transmission line is switched on, it takes some time for the current and voltage on the line to reach steady values. The transitional period is called the transient. The transient behavior just after closing the switch is usually analyzed in the frequency domain using the Laplace transform. For the sake of convenience, it will be treated here in the time domain. About one week of lectures are reserved to this material in the course.

Another required 300 junior-level microprocessor course entitled 'Microcomputer System Design and Applications' can benefit from the material presented here. In the Indiana University–Purdue University at Indianapolis (IUPUI) system this is EE361. The following materials are covered in the course [1–4]:

1. Introduction to the processors 8080/8085, 8088/8086; Motorola microprocessors; programming the microprocessor.
2. Building the microcomputer, part 1: the buses.
3. Building the microcomputer, part 2: adding memory.
4. Building the microcomputer, part 3: input/output.
5. Special-purpose support devices.
6. Serial I/O techniques.
7. Secondary storage techniques (floppy and rigid disc technology).
8. Microprocessor control applications.

The second part of this course covers some of the electrical characteristics of a bus: noise immunity, bus loading, reflections and bus-buffering techniques. About one lecture is reserved for the material.

Currently, for both digital electronics and electromagnetics, the materials needed consider only the resistive load transmission line. However, considering the capacitive component when modeling a load device such as memory or port units will have meaningful and accurate results. Also, in transmission stub matching systems, the model should include the reactive components of the matching line. Digital electronics texts, however, assume students have sufficient electromagnetics background to enable them to understand the theory behind the noise performance associated with transmitted signals and other topics like shielding, balancing and grounding taught in technical elective courses. Some undergraduate electromagnetics texts [5] cover briefly the transient performance to resistive load transmission lines. In this work, however, a general case with a resistive, capacitive or inductive load is being discussed.

The mismatching impedances and the reflection problems can be used on a graduate level as well. One application is the structure of composite materials where the different layers are to be made to maximize the power transmission across the different layers of a composite substrate [6]. More applications can be found in material engineering with composite substrate development.

The exact solution of the transient analysis using Maxwell's equations in the differential form applied to voltages and currents is quite complex. However, in most of the applications mentioned above, it is required to understand the step response when an electric pulse travels from a source point to a destination. It is more convenient to deal with such a problem using lattice diagrams—a technique of modeling the transient behavior of transmission lines. The transient analysis approach proposed here is integrated into a simple software program to be run on IBM machines and used in classrooms for junior and senior level courses.

Lattice, or bounce diagrams [7, 8], are used in transient analysis of transmission lines. In these diagrams, the quantities of interest are the incident and reflected waves from which the voltage and current can be obtained using superposition. Impedance mismatches cause multiple reflections on these lines, and time-varying input signals cause the voltage and current values to fluctuate with time. In this paper, the impedance mismatch is expressed in terms of reflection coefficients, and the time-varying impedance mismatch is expressed in terms of reflection coefficients, and the time-varying signals are not considered. All source voltages are considered as step or pulse inputs.

GENERAL TRANSMISSION LINE THEORY

For the general linear transmission line shown in Fig. 1, the voltage and current at any point, x, and any time t, can be expressed as:

$$\frac{\partial V(x,t)}{\partial x} = -\left[RI(x,t) + L\frac{\mathrm{d}I(x,t)}{\mathrm{d}t}\right] \quad (1)$$

and

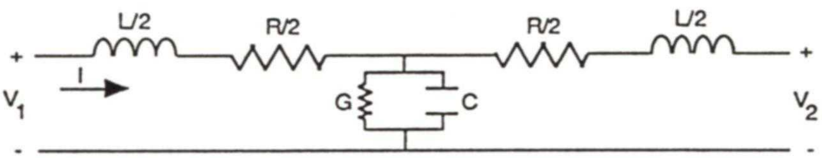

Fig. 1. General linear transmission line.

$$\frac{\partial I(x,t)}{\partial x} = -\left[GV(x,t) + C\frac{\mathrm{d}V(x,t)}{\mathrm{d}t}\right] \quad (2)$$

For a lossless transmission line, where R and G are zero, the above two equations become:

$$\frac{\partial V(x,t)}{\partial x} = -L\frac{\mathrm{d}I(x,t)}{\mathrm{d}t} \quad (3)$$

and

$$\frac{\partial I(x,t)}{\partial x} = -C\frac{\mathrm{d}V(x,t)}{\mathrm{d}t} \quad (4)$$

A general solution for equations (3) and (4) can be written as:

$$V(x,t) = V^{+}(x,t) + V^{-}(x,t) \quad (5)$$

and

$$I(x,t) = (1/R_0)[V^{+}(x,t) - V^{-}(x,t)] \quad (6)$$

where $V^{+}(x, t)$ represents the incident wave and $V^{-}(x, t)$ represents the reflected wave. R_0 represents the characteristic impedance of the lossless transmission lines which can be determined by:

$$R_0 = \sqrt{L/C} \quad (7)$$

In general, the characteristic impedance will be referred to as Z_0 and can have real and imaginary components.

The reflected waves mentioned above will be a function of the impedance of the source and the load. The effect of the source impedance is expressed as Γ_s, the source-side reflection coefficient, and the effect of the load impedance is expressed as Γ_l, the load-side reflection coefficient. They are determined by:

$$\Gamma_s = \frac{Z_g - R_0}{Z_g + R_0} \quad (8)$$

and

$$\Gamma_l = \frac{Z_l - R_0}{Z_l + R_0} \quad (9)$$

where Z_g and Z_l are the source and load impedances, respectively. In general, the quantities described in equations (8) and (9) are complex since Z_l and Z_g are in general complex quantities.

Assuming a lossless transmission line, the circuit used for analysis takes the form shown in Fig. 2. The starting voltage, V_s, is given by a simple voltage divider,

$$V_s = V_g\left(\frac{R_0}{R_0 + Z_g}\right) \quad (10)$$

and the starting current, I_s is

$$I_s = \frac{V_g}{R_0 + Z_g} \quad (11)$$

as the first incident waves will not be able to see the load impedance, Z_l. When these waves meet the load impedance, a portion will be reflected due to the impedance mismatch between R_0 and Z_l. The magnitude of this reflected voltage is

$$V_l^{-} = \rho_l \cdot V_s \quad (12)$$

Similarly, at the source side, the reflected wave will be created from the impedance mismatch between the source impedance and the characteristic impedance. This can be described as:

$$V_s^{-} = \rho_s \cdot V_l^{+} \quad (13)$$

In order to get the magnitude values for the voltage or current at the source or load sides at any time, t, all the reflected components are summed together.

RESISTIVE LOADED TRANSMISSION LINES

Figure 3 shows the transmission line terminated with a resistive load. The voltage V_l can be described as:

$$V_l = I \cdot R_l \quad (14)$$

In terms of the incident and reflected waves, the load voltage can be written as:

$$V_l = V^{+}(t) + V^{-}(t) \quad (15)$$

and

$$I = (1/R_0)[V^{+}(t) - V^{-}(t)] \quad (16)$$

Combining equations (14)–(16) gives:

$$V^{+}(t) + V^{-}(t) = (R_l/R_0)[V^{+}(t) - V^{-}(t)] \quad (17)$$

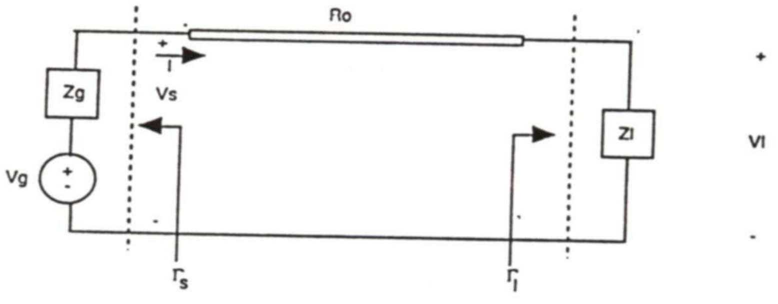

Fig. 2. Analysis circuit for lossless transmission line.

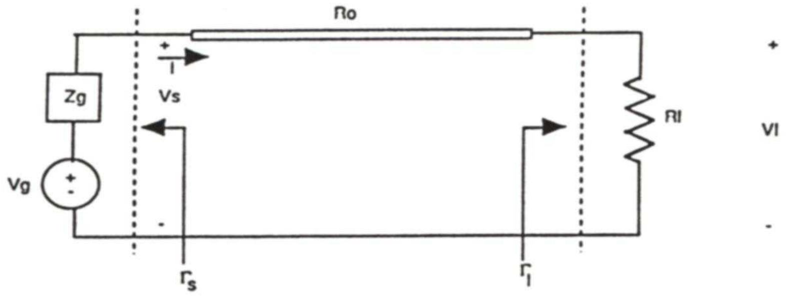

Fig. 3. Transmission line with resistive load.

With the aid of equation (9), the above expression can be rewritten as:

$$V^{-}(t)=\frac{R_l/R_0+1}{R_l/R_0-1}\,V^{+}(t)=\Gamma_l V^{+}(t) \quad (18)$$

A similar result is obtained when an incident wave falls at the source side. Instead of the wave being multiplied by Γ_l to produce the reflected wave, it is multiplied by Γ_s. In equation form,

$$V_s^{-}(t)=\Gamma_s\cdot v^{+}(t) \quad (19)$$

For the currents I_l and I_s, following equation (16), the reflected component at the load and source sides can be expressed as follows:

$$I^{-}(t)=-\Gamma_l\cdot I^{+}(t)\ \text{(load side)}$$

$$I^{-}(t)=-\Gamma_s\cdot I^{+}(t)\ \text{(source side)}$$

INDUCTIVE LOADED TRANSMISSION LINES

Figure 4 shows a lossless transmission line terminated with an inductive load. The voltage at the load side can be written as:

$$V_l=R\cdot I(t)+L\cdot\frac{\mathrm{d}(I)}{\mathrm{d}t} \quad (20)$$

where

$$\frac{\mathrm{d}[I(t)]}{\mathrm{d}t}=\frac{I(t+T)-I(t)}{T} \quad (21)$$

In equation (21), T is the time the waves take to travel from one end of the line to the other. It is calculated by dividing the length of the line, l, by the propagation velocity, v. The propagation velocity is a function of the inductance and capacitance of the lossless transmission line, and can be expressed as:

$$v=\sqrt{L\cdot C} \quad (22)$$

Combining equations (5), (6), (20) and (21), we get

$$V^{-}(t)=aV^{+}(t+T)+bV^{+}(t)+cV^{-}(t) \quad (23)$$

where the constants a, b and c are defined as:

$$a=\frac{L+RT-R_0T}{L+RT+R_0T}$$

$$b=\frac{-L}{L+RT+R_0T}$$

$$c=\frac{L}{L+RT+R_0T}$$

To find the value of $V\cdot(t+T)$, simply replace the parameter t by $(t+T)$ in equation (23) to get:

$$V^{-}(t+T)=aV^{+}(t+2T)+bV^{+}(t+T)+cV^{-}(t+T) \quad (24)$$

A series can be generated to calculate the nth order of the reflected waves as follows:

$$V^{+}(T)=V_s \quad (25)$$

$$V^{-}(T)=aV^{+}(T)+bV^{+}(0)+cV^{-}(0)=(a+b)V_s \quad (26)$$

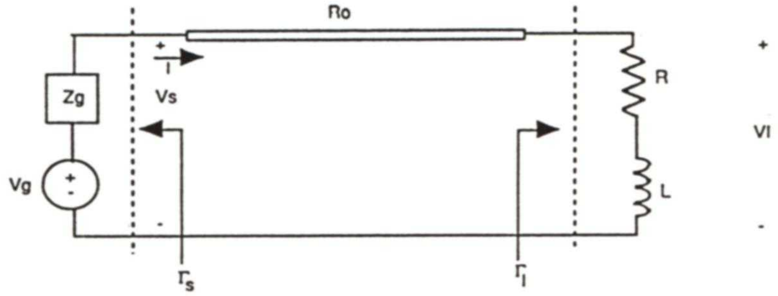

Fig. 4. Transmission line with inductive load.

and it follows that

$$V^+(3T) = V^-(T)\cdot\Gamma_s \quad (27)$$

Combining equations (24)–(27), $V^-(3T)$ can be written as:

$$V^-(3T) = (a+b)V_s[\Gamma_s(a+c)+b] \quad (28)$$

The above expression can be generalized to be in the nth order as:

$$V^-(nT) = (a+b)V_s[\Gamma_s(a+c)+b]^{(n-1)/2},\ n \text{ is odd} \quad (29)$$

The incident waves are:

$$V^+(T) + V^+(3T) + V^+(5T) + V^+(7T) + \ldots \quad (30)$$

The total load voltage resulting from the addition of the reflected and transmitted components can be calculated as:

$$V_{lt} = V_s + (1+\Gamma_s)\sum_{n=1,\text{odd}}^{N} V^-(nT) \quad (31)$$

where N is determined from T_f, the time the load voltage requires to reach steady state. T_f must be an odd multiple of T.

Combining equations (16) and (31), the load current can be obtained as:

$$I_{lt} = (1/R_0)\left[V_s + (1-\Gamma_s)\sum_{n=1,\text{odd}}^{N} V^-(nT)\right] \quad (32)$$

For the source side voltage,

$$V^+(0) = V_s \quad (33)$$

and

$$V^+(2T) = V^-(T) \quad (34)$$

Knowing that

$$V^-(t) = (a+b)V_s \quad (35)$$

then

$$V^+(2T) = (a+b)V_s \quad (36)$$

The reflected voltage, $V^-(2T)$, is

$$V^-(2T) = \Gamma_s V^+(2T) = \Gamma_s(a+b)V_s \quad (37)$$

Similarly,

$$V^-(4T) = \Gamma_s\{(a+b)V_s[\Gamma_s(a+c)+b]\} \quad (38)$$

Combining equations (33)–(38), the nth order reflected waves can be expressed as:

$$V^-(nT) = \Gamma_s(a+b)V_s[\Gamma_s(a+c)+b]^{(n-2)/2}\quad n \text{ is even} \quad (39)$$

The incident waves are:

$$V^+(0) + V^+(2T) + V^+(4T) + \ldots \quad (40)$$

The total source voltage includes the reflected and incident components and can be expressed as:

$$V_{st} = V^+(0) + V^+(2T) + V^-(2T) + V^+(4T) + V^-(4T) + \ldots \quad (41)$$

which can be rewritten as:

$$V_{st} = V_s + (1+\Gamma_s)\sum_{n=2,\text{even}}^{N} V^-(nT) \quad (42)$$

where N is a function of T_f, the time required by the source voltage to reach steady state. T_f is an even multiple of T, and must be greater than or equal to $2T$; as there is no reflection at 0, it is the source voltage itself, v_s.

CAPACITIVE LOADED TRANSMISSION LINES

Figure 5 shows a lossless transmission line terminated with a capacitive load. Using KVL, the load current can be calculated obtained as:

$$I_l = \frac{V_l}{R} + C\frac{dV_l}{dt} \quad (43)$$

and

$$\frac{dV_l}{dt} = \frac{V_l(t) - V_l(t-T)}{T} \quad (44)$$

Similar to the procedure followed in the case of inductive load, the voltages and currents can be rewritten in terms of reflected and incident wave, then combined to yield the following equations:

$$V^-(t) = a^{l}V^+(t) + b^{l}V^+(t-T0) + c^{l}V^-(t-T) \quad (45)$$

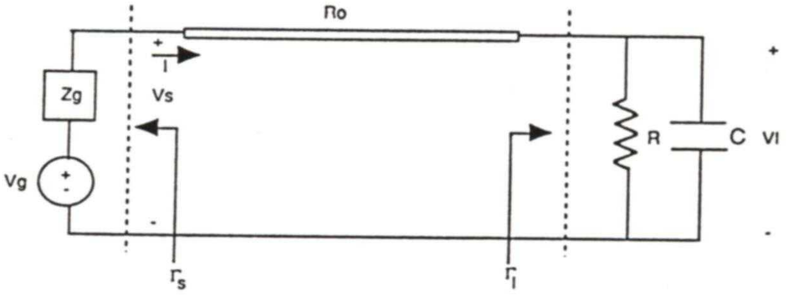

Fig. 5. Transmission line with capacitive load.

where the constants a^1, b^1 and c^1 are as follows:

$$a^1 = \frac{RT - Z_0T - RCZ_0}{RT + Z_0T + RCZ_0}$$

$$b^1 = \frac{-RCZ_0}{RT + Z_0T + RCZ_0}$$

$$c^1 = \frac{RCZ_0}{RT + Z_0T + RCZ_0}$$

These results indicate that the inductive and capacitive lattice diagrams are similar except for the voltage coefficients.

IMPLEMENTATION IN THE C PROGRAMMING LANGUAGE

This method of lattice diagram analysis was then implemented in the C programming language. The program, TRANS.EXE, was designed to analyze

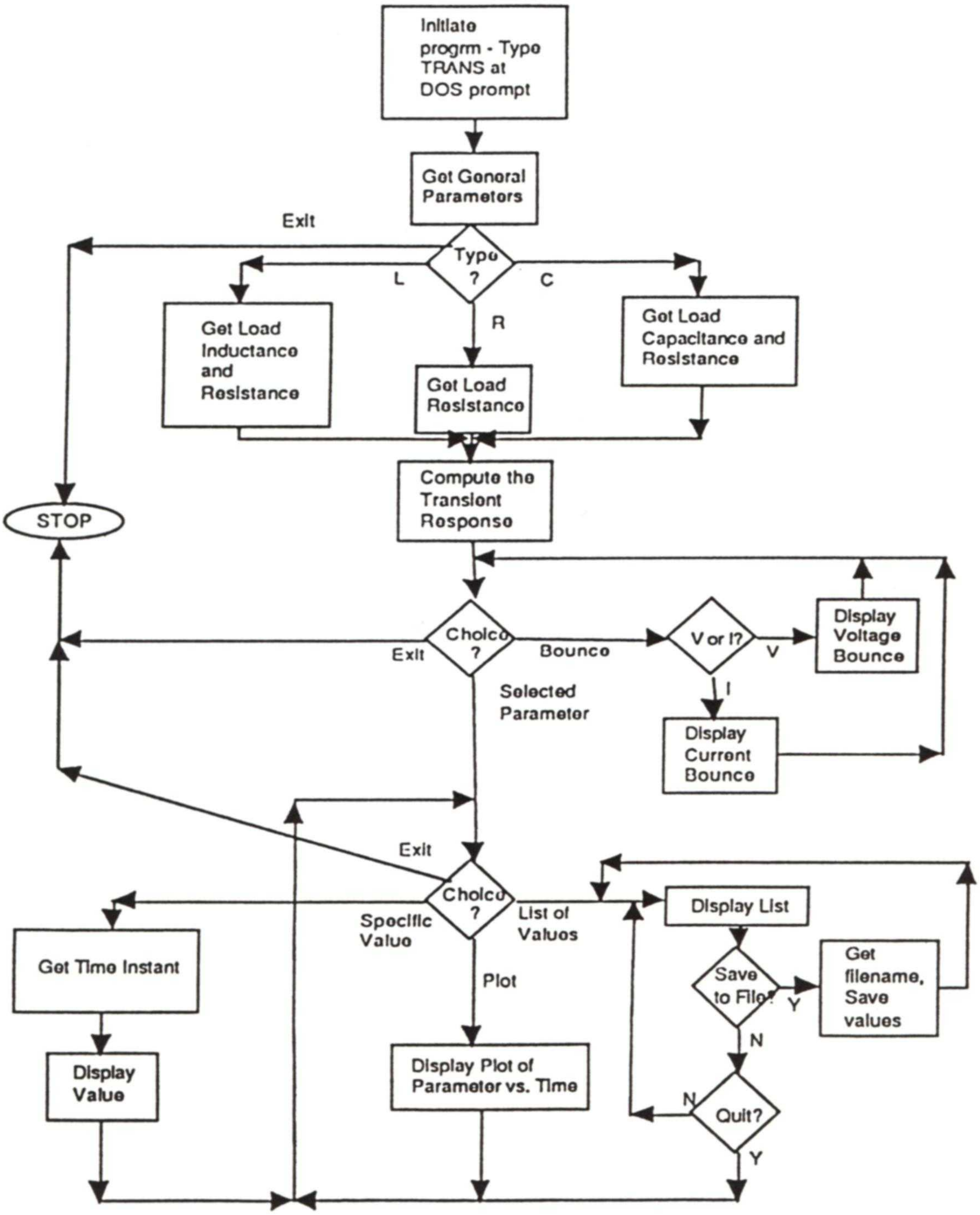

Fig. 6. Flow chart for lattice diagram analysis.

the three cases discussed above using a menu-driven format. This section describes the program constraints and flow of operation. The flow chart of the program is given in Fig. 6.

Program constraints

This program is written for an IBM PS/2 computer with 256K RAM and EGA graphics capability. The program must be run with the CAPS LOCK ON and all numbers entered must be in REAL number format. For example, to enter the number, 5, 5.0 would have to be keyed in.

The program will calculate the first 100 points of the transient response following the input of all parameters needed. These points are calculated every T time instants, where T is the time it takes the wave to propagate one length of the line. In other words, the voltage and current values are available only at the source and load ends of the transmission line.

Flow of operation

After initiating the program by typing TRANS at the DOS prompt, the user is presented with the main menu. Here, he or she must choose which type of transmission line is to be analyzed. The program takes care of all the array manipulation and transportation once the parameters are entered. Regardless of which circuit is analyzed, the parameters shown in Table 1 will always be needed, given their respective units. Depending on the circuit analyzed, the parameters shown in Table 2 may be requested.

After these parameters have been successfully entered, the program will calculate the transient response of the transmission line. Next, it will ask the user which parameter he would like to view. It will also be given in the option to view bounce diagrams or exit the program. The three existing options are:

Parameter selection. If the user selects one of the parameters, i.e. source voltage, source current, load voltage or load current.

Bounce diagram. If the bounce diagram option is selected, the user will be prompted for either the voltage or the current bounce diagram, and this will be displayed. The user will be returned to the same menu following the termination of the bounce diagram subprogram.

Quit. This option terminates the program.

If the user selected a parameter, the viewing menu is presented. This contains four options:

List of values. This option allows the user to view a listing of the selected parameter. The values can be saved and subsequently plotted. Pressing Q will return to the viewing menu.

Value at a specific time instant. This option allows the user to input a desired moment of time and obtain the value of the selected parameter. Following this, the user is returned to the viewing menu.

Paramter versus time plot. This option displays a graphical picture of the parameter selected by the user. Pressing any key returns the user to the viewing menu.

Select another parameter. This option returns the user to select a different parameter to view.

In this menu structure, in order to change the initial parameters that govern the circuit, such as the generator voltage, it is necessary to exit from the program and re-initiate it.

Table 1. Essential parameters

Parameter	Meaning	Units
Generator voltage	Voltage at the DC source	Volts
Generator resistance	Impedance of the DC source	Ohms
Line inductance	Lossless transmission line inductance	Henries
Line capacitance	Lossless transmission line capacitance	Farads
Line length	Lossless transmission line length	Meters

Table 2. Optional parameters

Parameter	Meaning	Units
Load resistance	Impedance of the load	Ohms
Load inductance	Inductance of the load	Henries
Load capacitance	Capacitance of the load	Farads

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