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On the Use of Statistical Design in Manufacturing Engineering

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This paper emphasizes the importance of teaching proper statistical design techniques in manufacturing engineering education. Metal cutting is chosen as an example area to outline the principles involved, and guidelines are provided to conduct and report experimental studies.

SUMMARY OF EDUCATIONAL ASPECTS OF THE PAPER

1. The paper discusses material for:
   (a) A complete course in Experimental Design for Manufacturing, *or*
   (b) Modules for use with a course in:
      (i) The Principles of Metalcutting, or
      (ii) Manufacturing Processes.
2. Students of the following departments could be taught this:

   Industrial Engineering
   Mechanical Engineering
   Production Engineering
3. Level of the course:

   Senior level (fourth-year undergraduate), or First-year graduate.
4. Mode of presentation:

   Lecture with discussion and a review of existing metal cutting literature. Also as instructions for laboratory experiments in manufacturing/metal cutting.
5. Is the material presented in a regular or in an elective course:

   The material should be presented as a module in a regular course, or in detail as a separate course on an elective basis.
6. Class hours required to cover the material (if treated as a module):

   2 hours if students have a background in experimental design.
   6 hours if students have no background in experimental design.
7. The material will be most useful when students have spent 5–6 hours evaluating current experimental metal cutting literature.
8. Description of novel aspects presented in the paper:

   This paper applies the principles of statistical design of experiments to experimental work in metal cutting so that valid conclusions can be drawn from the experiments. It also provides guidelines for conducting and reporting experimental studies.
9. Recommended Text:

   Metal Cutting by E. M. Trent, published by Butterworths, ISBN 0-408-10856-8.
10. This material is not covered at all in the text.

INTRODUCTION

A SUBSTANTIAL portion of engineering research is experimental in nature. The validity of conclusions drawn from experimental research is significantly influenced by the conditions under which the experiment is conducted, and therefore careful thought has to be given to designing the experiment properly. Researchers in the social and life sciences have long realized the importance of this fact. However, it has been our experience that the same is most often not true of engineering. As an illustration, most graduate engineering curricula do not even require a course in experimental design.

It is the intent of this paper to demonstrate the importance of teaching sound experimental design in manufacturing engineering education, and to motivate students and researchers in this area to incorporate these principles in their work. It is also intended to encourage graduate students who have an interest in any type of experimental research to study this subject as part of their education. The

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Table 1. Factors motivating experimental studies

FACTOR	EXAMPLE
A hypothesis to be tested for validity.	Vibrations generated by a worn cutting edge contain more high frequency energy than those generated by a sharp edge.
A functional relationship to be established between a response variable and a set of inputs in order to predict the response for the general case.	Taylor's Tool Life Equation.
An established theoretical result to be experimentally validated, when all the assumptions of the theoretical model are not exactly met.	Comparing experimental and theoretical values of the cutting gap in ECM for various electrolytes and gas densities.

principles involved are demonstrated by using the broad area of metal cutting as an example; however the principles apply equally well to any area involving experimental research.

In experimental research, the typical procedure would be to set up the required equipment, and to then make several experimental runs where observations are made on one or more variables of interest. The observations are then analyzed and appropriate conclusions drawn from the results of the analysis.

As an example, consider the general process of metal cutting that is basic to the study of manufacturing engineering. Operations in this area are often analyzed by conducting experiments where cutting and associated parameters are studied under various machining conditions. In general, a metal cutting study could be motivated by one or more of the factors shown in the first column of Table 1. The second column in the table gives an example from metal cutting for each case. In all cases, the common condition lies in the presence of a response variable of interest that is studied in the presence of one or more controlled inputs.

In general, as the inputs are changed, the levels of the observed responses also change. Variations in the observations could of course be due to a relationship between the response variable and the inputs. However, it could also be due to error, both error in measurement, and in random variations (or 'noise'). While it is difficult enough to draw conclusions about a system even when experimental data is relatively free of noise, the presence of random variations complicates this task tremendously. Under such circumstances it is critical for the experimenter to make use of some basic statistical techniques that can vastly improve the reliability of conclusions drawn.

By their very nature, the different areas of manufacturing are characterized by substantial variability. Metal cutting, for example, is characterized by high strain rates and a lack of external constraints during deformation. This complexity is often translated into relationships that can only be empirically captured. It also leads to significant interactions between material, cutting and tooling parameters. Thus scientific experimental design that accounts for these interactions is especially important in order to draw valid conclusions.

A detailed examination of the literature on experimental studies in the area of metal cutting displays an absence of statistical considerations in most cases. Even if such considerations are taken into account during a study, they are rarely if ever reported. The most striking aspect of these studies is that in most cases the conclusions drawn could

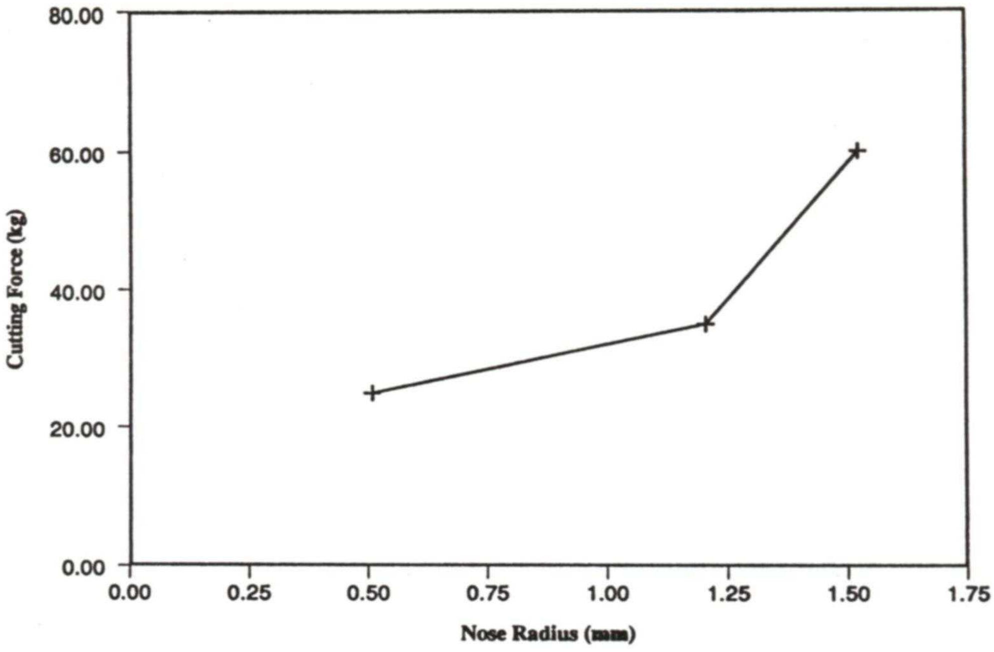

Fig. 1. Average cutting force as a function of nose radius.

have been made far more reliable and meaningful by expending the same amount of effort, but with more careful planning.

A good example of a study where a manufacturing experiment was carefully designed and executed, and the results analyzed using sound statistical considerations, is a tool wear experiment conducted and reported by Taylor [1]. Taylor's paper clearly demonstrates the importance of accounting for variability in an experimental study, and the use of appropriate statistical techniques in drawing conclusions. Unfortunately, reports such as these are few and far between.

In the following sections of this paper we detail why statistical methods must be integrated into manufacturing engineering curricula, the kinds of difficulties that they can mitigate (if not overcome) when conducting experimental work in this area, and provide guidelines on how an experiment should be conducted and reported so that the results can be interpreted meaningfully. Finally, we provide a possible outline for a course in experimental design for manufacturing.

TYPICAL DIFFICULTIES IN METAL CUTTING EXPERIMENTS

In investigating the relationship between a response and a set of inputs, an experimenter would typically encounter the following sources of difficulty [2]:

- presence of experimental error
- confusion of correlation and causation
- complexity of the effects of inputs

We now examine each of these a little further, in the context of a typical metal cutting experiment.

Consider a study [3] to determine the effect of tool nose radius on the forces (cutting, tangential, and radial) generated during a turning operation. Assume that there are three pieces of barstock of a given specification available for the study, and that each of these can yield six individual samples for actual machining; thus there is enough material for a total of 18 possible experimental runs.

Suppose standard carbide inserts are used and the experiment was conducted as follows: first, the six samples from the first piece of barstock were sequentially machined using a cutting tool of nose radius r_1, then the six from the second piece with a tool of nose radius r_2, and finally, the six from the third piece with a tool nose radius r_3 (say where $r_3 > r_2 > r_1$). In each of the 18 experimental runs, the cutting forces are measured using a transducer. The average of the cutting forces measured for each nose radius is then plotted against the appropriate nose radius. Suppose this results in the hypothetical graph shown in Fig. 1.

Looking at Fig. 1, one might be tempted to conclude that cutting forces increase nonlinearly with increase in cutting tool nose radius. However, if we relate this simple experiment with the three sources of difficulty listed above, a conclusion such as this could well be quite inaccurate.

Consider the first category of difficulties; this reflects the variations produced by factors other than those being directly considered in the study. The variations could be partly due to measurement error, but could also be caused by factors that are unknown or beyond the experimenters control; these have the effect of either obscuring the importance of certain factors or exaggerating the importance of others. In the nose radius study, the transducer could have an inherent measurement error. While it is possible that this error could be random, it could also be biased due to an oversight in calibration or due to a drift in the measuring device.

Even if the measurements were completely accurate, the observed variations may be caused by other factors. For instance, even though the three pieces of barstock are of the same specification, the hardness of each may be different because of batch variations during manufacture. There is no conclusive evidence to rule out the possibility that cutting forces are affected more by hardness than by the nose radius. Differences in other elements of the tool signatures of the inserts could be another factor; this is especially important in conducting studies using non-standard and custom designed tools.

The second category of difficulties relates to the ambiguity that often persists between causation and correlation. Causation refers to a *direct* cause and effect relationship between the response and the levels of an input. Correlation on the other hand refers to the situation where the response may vary with the levels of an input *not* because of any direct relationship, but rather, because they are both correlated through a common factor. This is an important distinction with a critical implication: a sound experimental study should look for causality and be capable of reducing the effect of correlation.

In the nose radius study, it is possible that the nose radius has only an indirect effect on the cutting force. The more relevant independent variable to be studied could perhaps be the total contact area between the tool and the workpiece. The contact area is a function of not only the nose radius, but also of other factors such as the depth of cut, the temperature of the tool-chip interface and the other elements of the tool signature.

The final source of difficulties is the complexity of the effects being studied. In many instances, the factors being employed in the study interact with one another. The implication is that the magnitude of the effect of a factor on the response variable under study depends on the level of some other factor; misleading conclusions can be drawn if this is not accounted for.

Suppose that in the nose radius study we also wish to check the effect of the depth of cut (d_j) on the cutting forces by treating it as a second independent variable. Suppose we denote the cutting force F_c with nose radius r_i and depth of cut d_j by $F_c(r_i, d_j)$. Then it may well be that, $F_c(r_1, d_1) - F_c(r_2, d_1)$ is not equal to $F_c(r_1, d_2) - F_c(r_2, d_2)$. This is because a higher depth of cut may mean a disproportionate increase in the frictional component of total energy expended during cutting.

Furthermore, the amount of experimental error that is transmitted may also be different at different levels of the factors. Finally, the response itself may behave in a highly nonlinear fashion with respect to various levels of a given factor. Statistically designed experiments have the attractive feature of actually trying to account for these complexities.

SOME GUIDELINES FOR CONDUCTING AND REPORTING MANUFACTURING BASED EXPERIMENTS

Many of the problems encountered in the course of a manufacturing investigation can be mitigated by the use of statistical techniques in designing and analyzing the experiment. This fact has been recognized more by researchers in the life sciences than in engineering, perhaps due to the fact that systems in the latter area have somewhat less inherent variability. Even if this were true, the benefits that can be accrued from sound statistical design principles, far outweigh the additional effort involved.

In this section, we summarize the typical procedures that an experiment should follow. The primary benefit from doing so is that this allows for a meaningful, appropriate and unbiased interpretation of the results. The three main principles of a statistically well-designed experiment are **replication**, **randomization** and **blocking**.

Replication refers to multiple observations for the same set of inputs. In general, a larger number of replicates results in more reliable conclusions. Randomization ensures that observations are independent of each other; this minimizes the effect of correlation. Finally, blocking is a procedure to make the experimental conditions more homogeneous so that the effect of extraneous variables is neutralized.

We now outline some general guidelines to be followed in designing and analyzing experiments.

Choice of Input Levels: The number of levels for each input in the study depends on the objective of the experiment and the feasibility of making multiple runs. In general, it is best to run a large number of replicates across a wide range of all inputs, but often this is not feasible. For example, metal cutting experiments tend to be time consuming and expensive. Thus one usually has to trade off the number of replicates against the number of different input levels. In general, if a functional relationship is to be evaluated it is better to use more levels spanning a wider range. On the other hand if a specific hypothesis is being tested, it is better to have more replicates.

Consider the nose radius study. It might have been better to have two replicates for each of nine different nose radii if a functional relationship between nose radius and cutting forces is to be established. Alternatively, six replicates at three levels might be adequate if we are just testing the hypothesis that nose radius has a significant effect on cutting forces (although in this case one should be careful about extrapolating any conclusions drawn to nose radii outside of the range spanned by the experiment).

Blocking: A block refers to a set of experimental conditions that are homogeneous and across which all input levels can be tested. Blocking tends to minimize the effect of extraneous variables. Blocks are often formed on the basis of time, batches of materials, or tools, since each of these presents opportunities for greater similarity. In the nose radius study, an example of blocking would be to assign two samples from each of the three pieces of barstock to each nose radius (rather than assigning all six from a single piece of barstock to a single nose radius). This would tend to neutralize the effect of any differences between different pieces of barstock.

Order of Runs: The order in which experiments are conducted should be completely random so that correlation effects are not present and thus the observations are independent of each other. In the nose radius study a possible procedure would be to first assign a specific nose radius to each of the 18 samples (based on blocking), and then decide on the actual order of the runs in a random fashion (e.g. by using a deck of 18 cards, or by drawing 18 slips of paper from a hat). This will tend to minimize the effect of measurement errors and other random sources of variation such as hard spots within a sample.

Interaction Effects: When more than one input variable (or factor) is studied, it is important to take into account the complexity of the effects. The interactions that may be present between the different factors could result in nonlinear effects on the response. These can be rigorously examined by means of an analysis of variance (ANOVA). While there are a number of excellent references on ANOVA, experimental studies in manufacturing engineering that actually report the use of this technique are relatively few. In general, we cannot emphasize enough the use of statistical techniques in making inferences. For the most part, the techniques are simple and go a long way in providing rigor to conclusions drawn from experimental studies.

Factorial Designs: A problem with statistical analyses where multiple factors are studied at several different levels is that a large number of experimental runs are required for legitimate conclusions. Suppose that we are studying the effects of five different elements of the tool signature on cutting forces. Each element is to be studied at two values (or levels). If a variable is at its first level it is assigned a code of − and if it is at its second level it is assigned a value of +. The scheme is summarized in Table 2.

Since we have a total of five factors and each factor is studied at two levels, we have a total of $2^5 = 32$ different combinations of factor levels. With two replicates for each of these, we would need a total of 64 runs. Such an experiment is called a *full factorial design* and is schematically represented in Table 3.

Now suppose we have material only for 32 runs, and we want to retain at least two replicates for each run. Statistical design techniques permit us to run experiments with *fractional factorial designs*. A half-fraction of the 2^5 design would use only 16 instead of all the 32 possible combinations. The choice of the levels for these 16 runs is critical and if done correctly, results in conclusions whose statistical validity approaches those from a full factorial design. If the levels are simply selected at random, then a statistical analysis of the results becomes impossible and any conclusions drawn from the experiment would have had limited validity. A valid half-fractional factorial design is shown in Table 4. For a detailed description of fractional factorial design, the reader is referred to references [2] and [4].

The use of fractional factorial designs has started to receive a lot of attention and Taguchi techniques that use orthogonal arrays are essentially based on the same concept.

Reporting Guidelines: When reporting the results of an experimental study we recommend that the following details be included:

- descriptions of the experimental setup
- levels and combinations of input variables, and the rationale for their selection
- number of replicates for each combination of inputs
- order in which runs were made
- an ANOVA table and statistical tests
- material characteristics (specification, hardness, etc.)
- any other information needed to replicate the data presented.

AN OUTLINE FOR A COURSE IN EXPERIMENTAL DESIGN FOR MANUFACTURING

A single semester course in experimental design for manufacturing could be designed to cover the following topics:

- brief review of basic statistics, hypothesis testing, and confidence intervals
- randomization, replication and blocking
- analysis of variance
- applications to manufacturing studies
- two-way factorial designs, Latin square designs
- general factorial designs
- fractional factorial designs
- review and critique of current literature in experimental manufacturing
- student project: Experimental study in manufacturing, incorporating statistical design.

CONCLUSIONS

This paper describes some of the problems associated with the design of engineering experiments. It also demonstrates how well-known statistical techniques can enable students and researchers in the area of manufacturing to greatly enhance the validity of conclusions from empirical data, with minimal additional effort. This points to

Table 2. Factor levels and codes for an experimental study to evaluate the effects of five elements of tool signature on cutting force (F_c)

Variables	Values used	
1 Nose Radius (mm)	r_1	r_2
2 Back Rake Angle (degrees)	α_1	α_2
3 Side Rake Angle (degrees)	β_1	β_2
4 End Relief Angle (degrees)	γ_1	γ_2
5 Side Relief Angle (degrees)	δ_1	δ_2
Level codes for values	-	+

Table 3. Full Factorial Design; Y_{ij} is the measured response (kg) for replicate j of run i (total of 64 measurements).

Run		Variable Levels					Response
No.	ID	1	2	3	4	5	Measured Cutting Force
1	1	-	-	-	-	-	Y_{11}, Y_{12}
2	2	-	-	-	-	+	Y_{21}, Y_{22}
3	3	-	-	-	+	-	Y_{31}, Y_{32}
4	4	-	-	-	+	+	Y_{41}, Y_{42}
5	5	-	-	+	-	-	Y_{51}, Y_{52}
6	6	-	-	+	-	+	Y_{61}, Y_{62}
7	7	-	-	+	+	-	Y_{71}, Y_{72}
8	8	-	-	+	+	+	Y_{81}, Y_{82}
9	9	-	+	-	-	-	Y_{91}, Y_{92}
10	10	-	+	-	-	+	$Y_{10,1}$, $Y_{10,2}$
11	11	-	+	-	+	-	$Y_{11,1}$, $Y_{11,2}$
12	12	-	+	-	+	+	$Y_{12,1}$, $Y_{12,2}$
13	13	-	+	+	-	-	$Y_{13,1}$, $Y_{13,2}$
14	14	-	+	+	-	+	$Y_{14,1}$, $Y_{14,2}$
15	15	-	+	+	+	-	$Y_{15,1}$, $Y_{15,2}$
16	16	-	+	+	+	+	$Y_{16,1}$, $Y_{16,2}$
17	17	+	-	-	-	-	$Y_{17,1}$, $Y_{17,2}$
18	18	+	-	-	-	+	$Y_{18,1}$, $Y_{18,2}$
19	19	+	-	-	+	-	$Y_{19,1}$, $Y_{19,2}$
20	20	+	-	-	+	+	$Y_{20,1}$, $Y_{20,2}$
21	21	+	-	+	-	-	$Y_{21,1}$, $Y_{21,2}$
22	22	+	-	+	-	+	$Y_{22,1}$, $Y_{22,2}$
23	23	+	-	+	+	-	$Y_{23,1}$, $Y_{23,2}$
24	24	+	-	+	+	+	$Y_{24,1}$, $Y_{24,2}$
25	25	+	+	-	-	-	$Y_{25,1}$, $Y_{25,2}$
26	26	+	+	-	-	+	$Y_{26,1}$, $Y_{26,2}$
27	27	+	+	-	+	-	$Y_{27,1}$, $Y_{27,2}$
28	28	+	+	-	+	+	$Y_{28,1}$, $Y_{28,2}$
29	29	+	+	+	-	-	$Y_{29,1}$, $Y_{29,2}$
30	30	+	+	+	-	+	$Y_{30,1}$, $Y_{30,2}$
31	31	+	+	+	+	-	$Y_{31,1}$, $Y_{31,2}$
32	32	+	+	+	+	+	$Y_{32,1}$, $Y_{32,2}$

Table 4. One half fractional factorial design with; Y_{ij} is the measured response (kg) for replicate j of run i (total of 32 measurements).

Run No.	Run ID	Variable Levels 1	2	3	4	5	Response: Measured Cutting Force
1	2	–	–	–	–	+	Y_{11}, Y_{12}
2	3	–	–	–	+	–	Y_{21}, Y_{22}
3	5	–	–	+	–	–	Y_{31}, Y_{32}
4	8	–	–	+	+	+	Y_{41}, Y_{42}
5	9	–	+	–	–	–	Y_{51}, Y_{52}
6	12	–	+	–	+	+	Y_{61}, Y_{62}
7	14	–	+	+	–	+	Y_{71}, Y_{72}
8	15	–	+	+	+	–	Y_{81}, Y_{82}
9	17	+	–	–	–	–	Y_{91}, Y_{92}
10	20	+	–	–	+	+	$Y_{10,1}$, $Y_{10,2}$
11	22	+	–	+	–	+	$Y_{11,1}$, $Y_{11,2}$
12	23	+	–	+	+	–	$Y_{12,1}$, $Y_{12,2}$
13	26	+	+	–	–	+	$Y_{13,1}$, $Y_{13,2}$
14	27	+	+	–	+	–	$Y_{14,1}$, $Y_{14,2}$
15	29	+	+	+	–	–	$Y_{15,1}$, $Y_{15,2}$
16	32	+	+	+	+	+	$Y_{16,1}$, $Y_{16,2}$

a strong need for including a course in applied experimental design for students of manufacturing engineering. It is hoped that this paper will also motivate researchers to use the abundance of literature on statistical design in future experimental studies in engineering.

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