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The Automatic Computation of Influence Lines

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This paper presents an application of numerical analysis for the determination of influence lines. By using Land's theorem, this method shows the influence lines for actions, such as bending moment and shear, in a completely automatic way. Almost all of the existing numerical codes for the solution of frame structures can be used.

SUMMARY OF EDUCATIONAL ASPECTS OF THE PAPER

1. The paper discusses material for a course in: Bridge Constructions, Automatic Calculus of Structures.
2. Students of the following departments are taught in the course: Civil Engineering.
3. Level of the courses: Fifth year undergraduate Bridge Constructions (in Italy) Fourth year undergraduate Automatic Calculus of Structures (in Italy)
4. Mode of presentation: Exercises, applications to actual structure calculation, discussion group.
5. Is the material presented in a regular or in an elective course: The material is presented in regular courses (Bridge Constructions and Automatic Calculus of Structures) of the Civil Engineering Degree Course.
6. Class hours required to cover the material: Four classroom hours are required.
7. Student homework and revision hours required for the material: Five hours of student homework and two hours of revision.
8. Description of the novel aspects presented in the paper: The method described in the paper presents the influence lines of any structure (e.g. bridges) in a completely automatic way, by using almost all the existing numerical code for the solution of frame structures.
9. The standard text recommended for the course, in addition to authors' notes:
   [1] V. Franciosi, *Scienza delle Costruzioni*, Liguori Napoli (1962).
   [2] J.B. Kennedy and M.K.S. Madugla, *Elastic Analysis of Structures, Classical and Matrix Method*, Harper & Row, New York (1990).
10. The material is/is not covered in the text. The discussion in the text is different in the following aspects: In [1] and [2] there is no discussion on the possibility of using numerical code to calculate the influence lines. [1] is a good text to study the theoretical meaning of influence lines and to better understand the application of Land's theorem presented in the paper. [2] is a useful book on classical and matrix methods to solve elastic structures, with an extensive section on the influence lines determination.

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LIST OF SYMBOLS

- M_s^* is the bending moment that arises in the cross-section S due to the imposition of the settlements of the supports at the left-hand side of section S itself;
- T_s^* is the shear that arises in the cross-section S due to the imposition of the settlements of the supports at the left-hand side of section S itself;
- Y^* is the vertical displacement of the beam resulting from the elastic numerical analysis;
- z is the distance between the generic section and the section S;
- R is the reaction of the support;
- φ_r is the relative cross-section rotation;
- u_r is the relative displacement tangent to the axis of the arch;
- v_r is the relative displacement perpendicular to the axis of the arch;
- Y' is the vertical component of the hinge translation;
- H is the horizontal thrust of the arch;

Y''' is the vertical component of the initial translation of the considered point, that is generally different from the vertical component of the initial translation of the hinge Y'.

INTRODUCTION

THE EFFECT of live or of moving dead loads can be effectively analyzed and simply represented in graphical form by the use of influence lines. The concept of influence lines was first introduced by Winkler in 1868 and then studied by several authors [1, 2, 3, 4]. An influence line can be defined as the representation of the variation of a particular action such as reaction, shear, bending moment, deflection at a particular section or axial force in a particular member, plotted against the position of a unit point load moving across the structure [4]. For a proper design of some particular structures it is very useful to know the shape of the influence lines that show which parts of the structure should be loaded to obtain maximum effects. For example, when a truck gets through a bridge, the forces in the members vary with the position of the truck and a knowledge of the influence lines enables the critical position of the load producing the maximum stress in any specified member to be found.

Difficulties can arise for statically indeterminate structures [2], for which the influence lines are composed of curves and therefore several ordinates must be computed, requiring the analyses of many load conditions for every considered cross-section. Problems can arise also for arches [1, 2], where the curvature of the structure makes the tracing of influence lines more complex [1].

In this paper we deal with the method of obtaining influence lines for statically indeterminate structures in a wholly automatic way. Both continuous beams and arches can be considered. The method is based on Land's theorem [1, 5, 6, 7], and can be applied by using most of the existing numerical codes developed to solve frame structures. Some applications with a microcomputer program are presented to demonstrate the simplicity and the effectiveness of this method.

INFLUENCE LINES FOR CONTINUOUS BEAMS

Shear for a cross-section in a middle span

Let S be the cross-section for which the shear influence line must be traced (Fig. 1). By imposing a unit vertical displacement to the supports at the left-hand side of section S, a bending moment M_s^* and a shear T_s^* arise in the cross-section S itself. The deformed beam is now cut at S, keeping M_s^* and T_s^* applied at the end of the two parts of the beam, and the supports are put back in their original positions. As a consequence, two deformed curves are obtained that represent exactly the influence line for the shear. In fact Land's theorem conditions are respected (the relative rotation and horizontal displacement are zero) and we can write

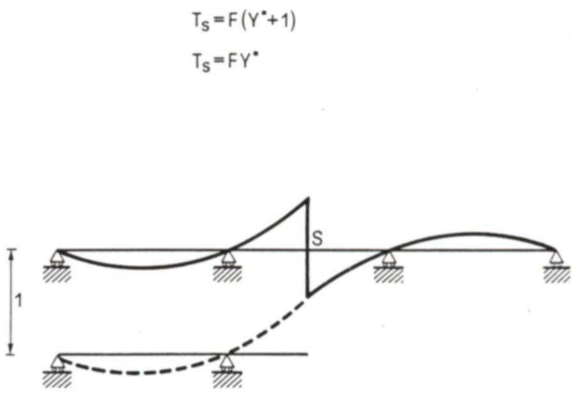

Fig. 1. Influence line for shear in the middle span of a continuous beam

$$T_s = F(Y^* + 1)$$

for the left-hand side of the beam, with respect to S and

$$T_s = FY^*$$

for the right-hand side of the beam, with respect to S, where Y^* is the vertical displacement of the beam resulting from the numerical analysis. Since many automatic codes provide only the node displacements, it is useful to introduce some fictious nodes to obtain a more precise tracing of influence lines.

Shear for a cross-section in the extreme span

To derive the influence line for the shear at the cross-section S (Fig. 2), a unit vertical displacement is assigned to the extreme support and a cut is executed at S. With reference to Fig. 2, even if the cut turns the left-hand side of the beam into a mechanism, the presence of T_s^* and M_s^* makes this structure in equilibrium, so proceeding as in the previous case:

$$T_s = F(Y^* + 1)$$

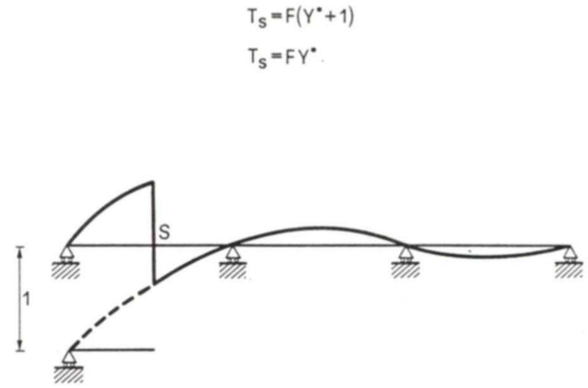

Fig. 2. Influence line for shear in the extreme span of a continuous beam

for the left-hand side of the beam, with respect to S and

$$T_s = FY^*$$

for the right-hand side of the beam, with respect to S, where Y^* has the previous meaning. It is worth noting that just one load condition (i.e. a support settlement) is necessary to obtain the influence line for the shearing force for every cross-section belonging to the same span. This is because the deformed configuration is always the same for all the sections of a span.

Bending moment for a cross-section in a middle span

A vertical displacement is assigned to the supports at the left-hand side of the beam, with respect to section S (Fig. 3), so that a unit rotation around S is obtained. With reference to Fig. 3, the unit rotation around the section S is obtained by imposing a vertical displacement $v = z_1$ to the extreme support (z_1 is the distance between the extreme support and the section S), and a vertical displacement $v = z_2$ to the second support (z_2 is the distance between this support and the section S).

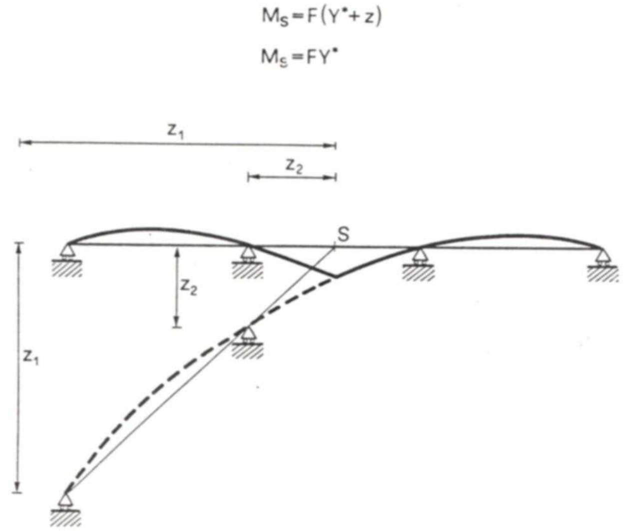

Fig. 3. Influence line for bending moment in the middle span of a continuous beam

After deformation, the beam is cut at S and the supports are put back in the original positions, by keeping the bending moment M_s^* and the shear T_s^* applied. Again, Land's theorem conditions are respected and we can write:

$$M_s = F(Y^* + z)$$

for the left-hand side of the beam, with respect to S and

$$M_s = FY^*$$

for the right-hand side of the beam, with respect to S, where Y^* is again the vertical displacement of the beam resulting from the numerical analysis, and z is the distance between the generic section and the section S (Fig. 3).

Bending moment for a cross-section in the extreme span

The influence line for the bending moment at S is obtained as in the previous case and is shown in Fig. 4. Again we have:

$$M_s = F(Y^* + z)$$

for the left-hand side of the beam, with respect to S and

$$M_s = FY^*$$

for the right-hand side of the beam, with respect to S, where Y^* has the previous meaning.

It should be noted that to trace the influence line for the bending moment it is necessary to apply one load condition for every considered cross-section, even if they belong to the same span. This is because the deformed configuration is different from section to section, also in the same span.

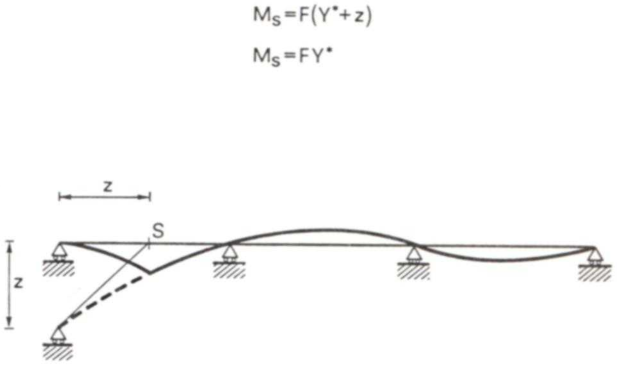

Fig. 4. Influence line for bending moment in the extreme span of a continuous beam

Reaction of a support

Finally, to obtain the influence line for the vertical reaction R of a support, it is sufficient to assign a unit vertical displacement to the support itself. The deformed curve obtained by the numerical analysis is exactly the influence line for the reaction with $R = FY^*$.

INFLUENCE LINES FOR TWO HINGED ARCHES

Shear for vertical loads

With reference to Fig. 5, let φ_r be the relative cross-section rotation and u_r and v_r be the relative displacements respectively tangential and perpendicular to the arch axis. The hinge at the left-hand side of section S is translated in a parallel manner to the inclination of the section S itself. Then the arch is cut at S and the left hinge is moved to its original position by obtaining:

$$\varphi_r = 0; \qquad u_r = 0; \qquad v_r = 1.$$

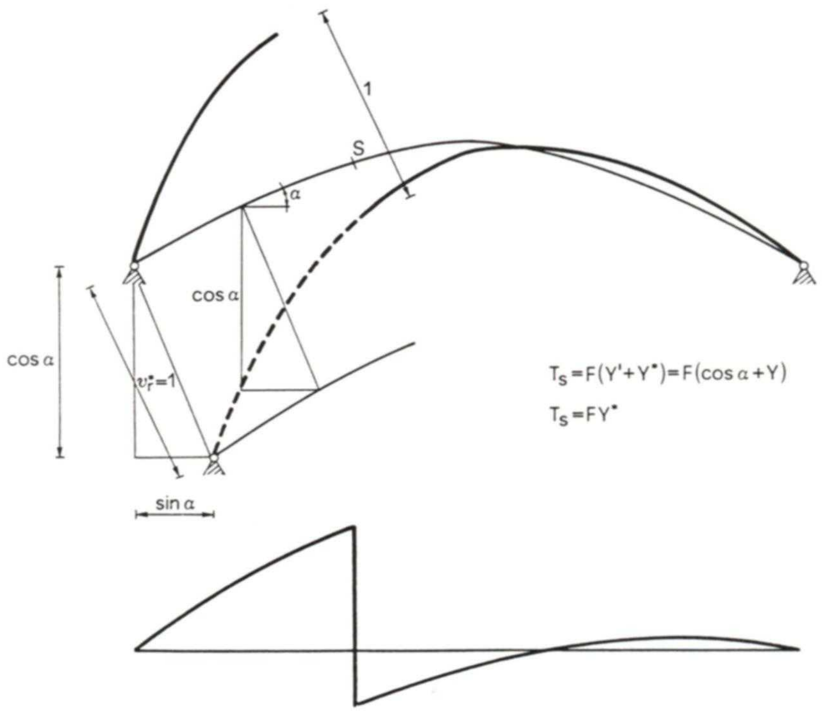

Fig. 5. Influence line for shear in a two hinged arch

Hence the shearing force can be written as:

$$T_s = F(Y^* + Y')$$

for the left-hand side of the beam, with respect to S and

$$T_s = FY^*$$

for the right-hand side of the beam, with respect to S, where Y' is the vertical component of the hinge translation and Y^* has the previous meaning. To obtain the influence line (Fig. 5), the $(Y^* + Y')$ and Y^* values must be plotted as ordinates on a straight beam with length equal to the span of the arch (reference beam).

Horizontal thrust for vertical loads
A unit translation of the left hinge is assigned in the direction parallel to the span of the arch, and the vertical displacements resulting from the numerical analysis are then plotted as ordinates on the reference beam (Fig. 6) to obtain the influence lines for the thrust: $H = FY^*$.

Bending moment for vertical loads
With reference to Fig. 7, the hinge at the left-hand side of section S is moved so that the left part of the arch rotates round S of a unit quantity. Next, the arch is cut at S and the opposite rotation is assigned to the left hinge by obtaining:

$$\varphi_r = 1; \qquad u_r = 0; \qquad v_r = 0.$$

Hence we can write (by Land's theorem):

$$M_s = F(Y^* + Y'')$$

for the left-hand side of the beam, with respect to S, and

$$M_s = FY^*$$

for the right-hand side of the beam, with respect to S, where Y'' is the vertical component of the initial translation of the considered point, that is generally different from the initial, vertical component of the hinge translation, and Y^* has the previous meaning. Again $(Y^* + Y'')$ and Y^* are plotted as ordinates on the reference beam (Fig. 7) to obtain the influence line for the bending moment.

FIXED ARCHES

The method is the same used in the previous section, except for the influence line for the bending moment. In this case, when we assign the displacement of the support, the fixed section must rotate of a unit value, as shown in Fig. 8.

It is worth observing that the arches can be schematized with straight lines if the microcomputer programs for frame structures do not consider curvilinear beams.

CONCLUSION

To improve and simplify the use of influence lines that are very helpful in the analysis of structures subjected to live or moving dead loads, it is

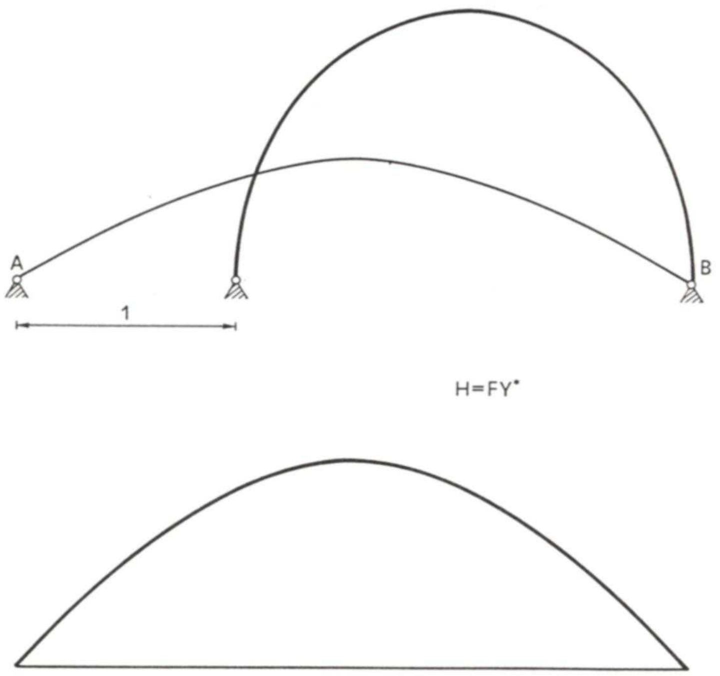

Fig. 6. Influence line for thrust in a two hinged arch

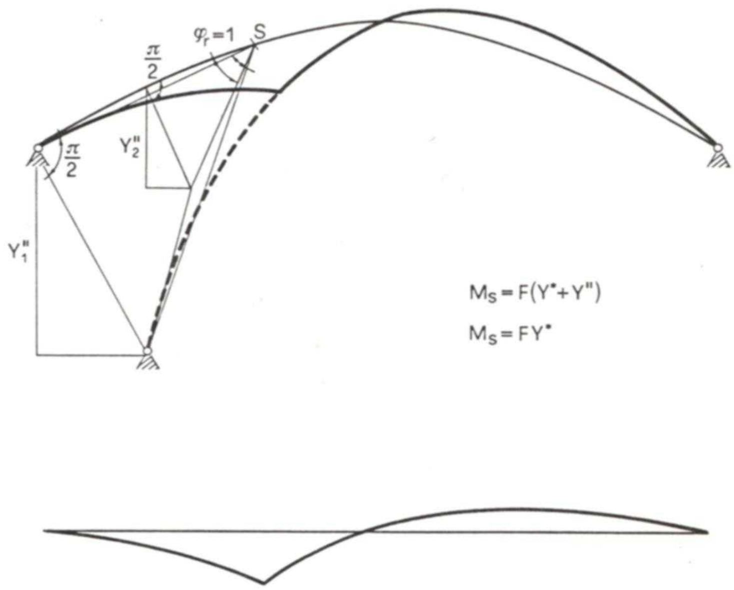

Fig. 7. Influence line for bending moment in a two hinged arch

necessary to automatize the procedure for obtaining the line drawings. In particular, for statically indeterminate structures, the tracing of influence lines without an automatic aid can become an onerous task.

The method described in this note is very simple and effective and produces the influence lines for every action, just using a generic automatic code for frame structures and imposing suitable load conditions, i.e. support settlements, so that Land's theorem hypotheses are respected.

For the sake of brevity, only a few particular cases have been presented, concerning continuous beams and arches. However, it is easy to apply this method to any structure to obtain the influence lines for every action that varies as the load moves across the structure, e.g. bending moment, shearing force, thrust, displacement at a section or reaction at a support.

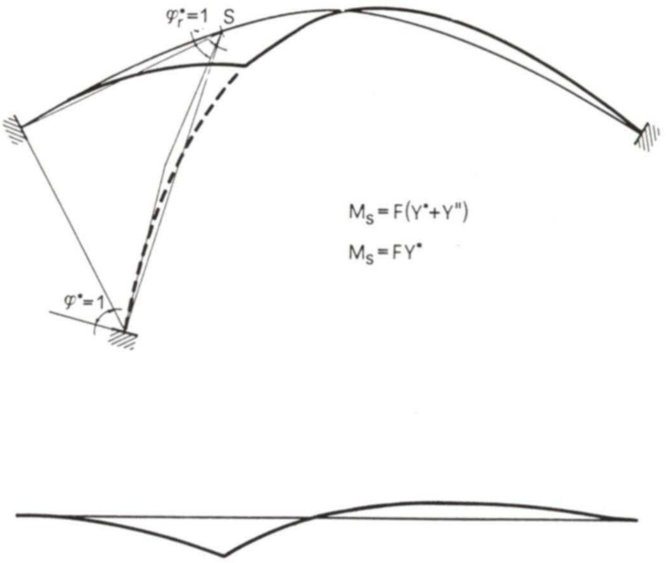

Fig. 8. Influence line for bending moment in a hingeless arch

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