*Int. J. Engng Ed.* Vol. 8, No. 3, pp. 224–227, 1992
Printed in Great Britain.

0742–0269/91 $3.00+0.00


# Fatigue of a Demonstration Rig for Unbalanced Reciprocating Forces*

J. H. ONG

*Nanyang Technological University, School of Mechanical and Production Engineering, Nanyang Avenue, Singapore 2263*

*In the past, an engineering undergraduate demonstration rig which is used primarily for observing the effects of unbalanced reciprocating forces usually failed in fatigue within a period of approximately 10 weeks. This paper outlines the solution to the problem by redesigning the rig with a new cantilever moulded with an araldite base. The present set-up is still in good working order after a period of two years.*

## INTRODUCTION

THE PROBLEM of fatigue on an engineering undergraduate demonstration rig is studied in this paper. The rig [1] as shown in Fig. 1 is a model four-cylinder in-line engine-electric motor assembly mounted at the end of a single cantilever of circular cross-section with the centre of mass of the assembly on the axis of the cantilever. The set-up is primarily used by the undergraduate students to observe the effects of unbalance of primary or secondary forces and couples in a four-cylinder in-line engine.

At the Nanyang Technological Institute, the rig is used three afternoons per week by different groups of students. The duration of each laboratory session per group of students is three hours. The original cantilever as shown in Fig. 2(a) tends to break after a period of approximately 10 weeks.

The problem was solved by redesigning the cantilever. The new design, as shown in Fig. 2(b), is a combination of a steel bar with an araldite-moulded base for the clamped end. With this new design, the present set-up which was implemented two years ago is still in good working order.

## CAUSE OF THE PROBLEM

If the engine is out-of-balance, the out-of-balance forces and couples will act as an exciting force on the cantilever. If the engine is now running at a speed equal to one of the natural frequencies of the system, the system will vibrate violently due to resonance. The amplitude of vibration is a function of the magnitude of the out-of-balance force or couple. The magnitude of the out-of-balance force or couple [2, 3] is directly related to the crank angle setting and the forces required to accelerate the pistons. The force $F$ required to accelerate a piston is given by:

$$F = Mr\omega^2\left(\cos\theta + \frac{1}{n}\cos 2\theta\right) \qquad (1)$$

where $M$ is the mass of the reciprocating mass, $r$ is the crank radius, 1 the length of the connecting rod, $\omega$ the crank speed and $\theta = \omega t$.

In the laboratory sessions, to observe and analyse the effects of the unbalanced forces, the students usually run the engine at resonance. The primary cause of the breakage in the cantilever is due to the primary exciting forces for vertical oscillation of the system.

## SOLUTION TO THE PROBLEM

The service life of the cantilever is directly related to the alternating strain amplitude at the root of the cantilever and can be characterized by a strain-life curve [4, 5] as shown in Fig. 3. The strain-life curve as shown in Fig. 3 can be expressed as:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \qquad (2)$$

where $\Delta\varepsilon/2$ is the strain amplitude, $2N_f$ the reversals to failure, $E$ is Young's modulus, $\sigma_f'$ the fatigue strength coefficient, $b$ the fatigue strength exponent, $\varepsilon_f'$ the fatigue ductility coefficient and $c$ the fatigue ductility exponent. $\sigma_f'$, $b$, $\varepsilon_f'$, and $c$ are the material fatigue properties of the cantilever.

As can be seen from the strain-life expression, the service life of the cantilever will be increased if the alternating strain amplitude at the root of the cantilever is reduced. The alternating strain amplitude can be reduced by considering the following three factors in the redesigning of the cantilever. The factors are as follows:

(i) Reducing the stress concentration factor at the clamped end.
(ii) Reducing the first natural frequency of the system for vertical oscillation. Thus, when the

* Paper accepted 10 April 1992.

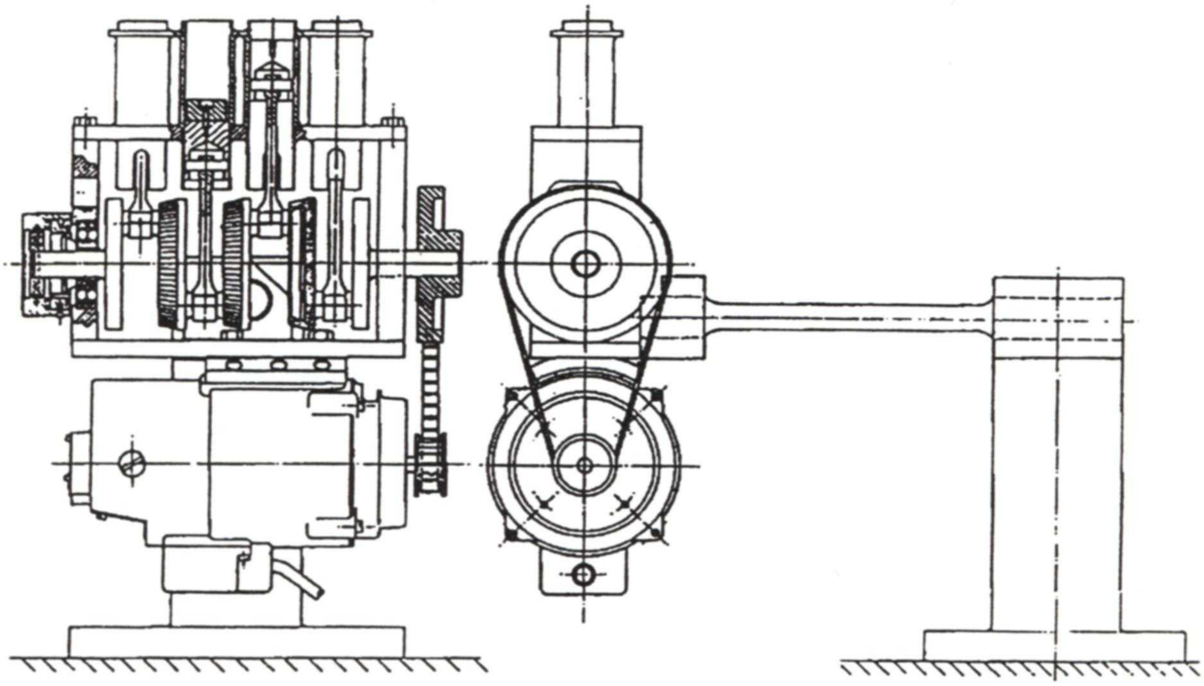

Fig. 1. Experimental rig—a model four-cylinder in-line engine

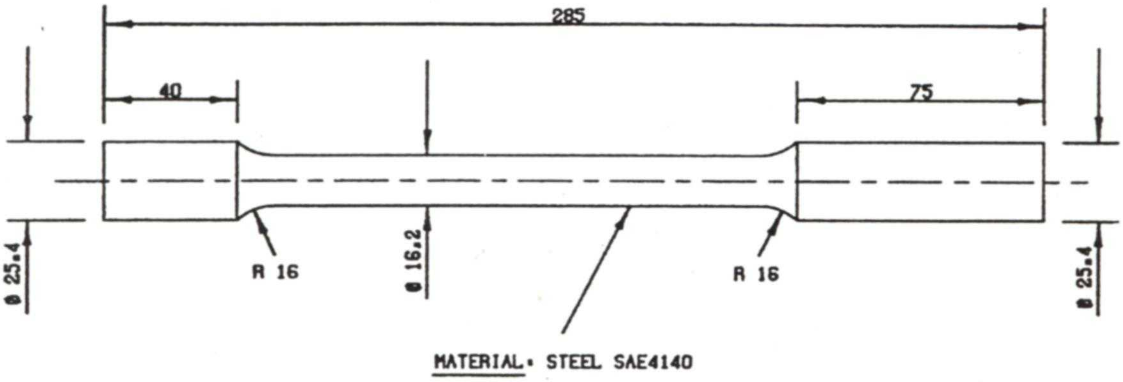


Fig. 2(a). The original steel cantilever

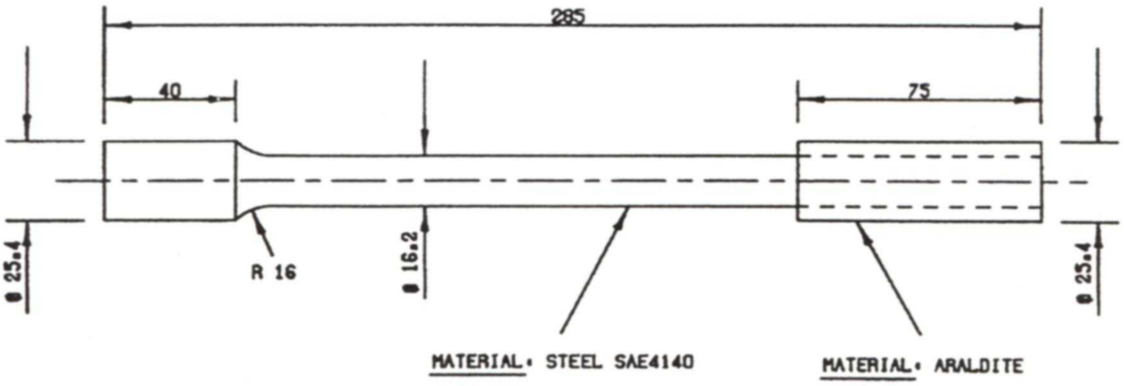


Fig. 2(b). The redesigned cantilever with moulded araldite end

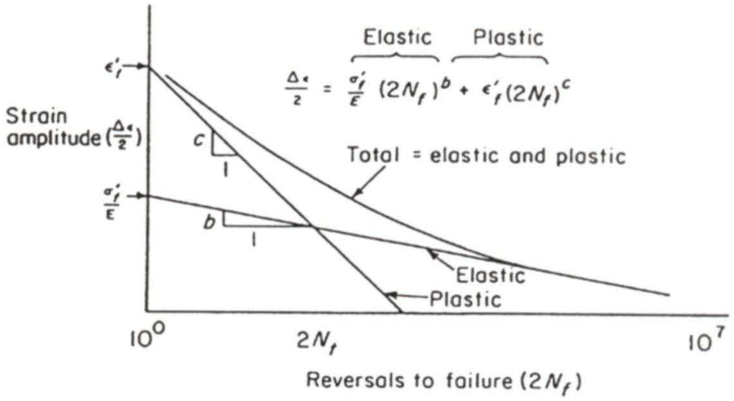


Fig. 3. Typical strain-life curve

students run the engine at resonance, the out-of-balance forces will be reduced as the unbalanced forces are directly proportional to the square of the engine speed, $\omega$.

(iii) Reducing the dynamic magnification or gain of the system at resonance. At resonance, the gain is equal to $1/2\xi$ where $\xi$ is equal to the proportional damping ratio at that particular natural frequency. Thus, increasing the proportional damping ratio $\xi$ will effectively reduce the dynamic magnification factor for the response of the system.

## THE REDESIGNED CANTILEVER

The redesigned cantilever is shown in Fig. 2(b). It has a moulded araldite base for the clamped end of the cantilever. With this design, there will be no stress raiser at the clamped end. This is due to the fact that the Young's moduli of araldite and steel are approximately 3.45E9 N/m$^2$ and 209E9 N/m$^{-2}$, respectively. Thus, the stiffness of araldite is approximately equal to 1/60 of the stiffness of the steel bar. Hence, the effect of the araldite base on the main solid steel bar in producing a stress raiser is negligible.

The moulded araldite base is very much softer than the rigidly mounted clamped end condition for the original cantilever. This is equal to a relaxation of the boundary condition of a rigidly mounted end to one of a spring support. Hence, according to Rayleigh's principle, the natural frequencies of the system will be reduced accordingly. With the exception of the base, the dimensions of the steel portion of the redesigned cantilever are kept the same as the original cantilever. In doing so, the natural frequencies of the system are reduced.

The value of hysteretic damping factor are very variable. However, to get a measure of the degree of structural damping, the hysteretic damping factors quoted in the PAFEC-Data Preparation Manual [6] for steel and araldite are 0.005 and 0.05, respectively. Since the damping factor for araldite is 10 times larger than that of steel, the redesigned cantilever with the araldite moulded base will definitely have a much lower dynamic magnification factor as compared to the original cantilever which is made completely out of steel.

## PREPARATION OF THE REDESIGNED CANTILEVER

A steel shaft was machined according to the dimensions given in Fig. 2(b). The end which is to be casted with the araldite was knurred so as to have better bonding between the steel and araldite. Preparation of the araldite mixture was according to the instructions given by the supplier CIBA-GEIGY S.E. ASIA Pte Ltd. A piece of aluminium pipe section of 35 mm diameter and 85 mm length was used for the moulding process. The inner surface of the aluminium pipe was greased so as to prevent sticking. The shaft was then placed into the aluminium pipe section and supported in a vertical position by a retock stand. The araldite mixture was then poured and left to cure over a period of 24 hours. After curing, the aluminium section was eased out and the moulded end was machined to specifications.

## ANALYSIS OF ORIGINAL/REDESIGNED CANTILEVERS

The original/redesigned cantilevers are made from the same steel bar with material specified as SAE 4140. The cantilevers were installed into the demonstration rig as shown in Fig. 1 and were then tested at their respective first natural frequency for vertical oscillations. For the tests, the conventional flat crankshaft layout was used, i.e. the crank angles were set relative to the outboard cylinder at 0°, 180°, 180° and 0°. The tests simulate the damage process commonly encountered in a normal laboratory session as conducted by the undergraduate students. The results of the tests are shown in Fig. 4.

Figure 4 shows that the strain at the root of the cantilever is dramatically reduced for the redesigned shaft. Based upon the measured strains as shown in Fig. 4, the predicted lives to failure using

| | ORIGINAL SHAFT | REDESIGNED SHAFT |
|---|---|---|
| Natural frequency(rpm) for vertical oscillations | 811 | 710 |
| Alternating strain amplitude at root of the cantilever due to 2nd order out of balance forces at the first bending frequency of the system | 2409μ | 759μ |
| Proportional damping factor (ξ) | 0.008 | 0.016 |
| Number of predicted cycles to failure | $1.215*10^7$ | $1.667*10^{13}$ |
| Equivalent no of operating hours to failure = cycles to failure/first bending frequency | 249 hrs | $3.913*10^8$ hrs |

Fig. 4. Experimental/predicted data

equation (2) are 249 hours and $3.913 \times 10^8$ hours for the original shaft and the redesigned shaft, respectively. Hence, it is not surprising to find that the redesigned shaft with the moulded araldite end is still in good working order after a period of two years.

## CONCLUDING REMARKS

1. The redesigned cantilever with the moulded araldite base is simple to make and helps to improve the fatigue life of the demonstration rig dramatically.
2. The fatigue life of the cantilever is governed by its material fatigue properties, i.e. fatigue strength coefficient, fatigue strength exponent, fatigue ductility coefficient and fatigue ductility exponent. By choosing steel with appropriate material fatigue properties, it is possible to further enhance the fatigue life of the redesigned cantilever.
3. With the redesigned cantilever, the present demonstration rig which was implemented two years ago is still in good working order. Prior to the change, the cantilever shaft usually failed within a period of 10 weeks.

## REFERENCES


1. *TecQuipment TM22 Balancing of Reciprocating Masses Manual*, TecQuipment Ltd., Nottingham, England.
2. J. L. M. Morrison and B. Crossland, *An Introduction to the Mechanics of Machines*. Longland Group Ltd., London, pp. 204–225 (1978).
3. J. Hannah and R. C. Stephens, *Mechanics of Machines—Advanced Theory and Examples*. Edward Arnold (Publishers) Ltd., London, pp. 159–187 (1982).
4. N. E. Dowling, Fatigue life prediction for complex load versus time histories. *Trans. ASME J. Eng. Materials Technol.*, **105**, 206–214 (1983).
5. J. H. Ong, Fatigue-life predictions on microcomputers. *Comp.-Aid. Des.*, **19**(2), 95–101 (1987).
6. R. D. Henshell, *PAFEC75—Data Preparation Manual*. PAFEC Ltd., Nottingham, U.K.