An Interdisciplinary Approach to Energy Conversion*

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In science, as well as in engineering, the concept of analogy has proved a very effective tool for information transfer, easing the introduction of new concepts and ideas in an interdisciplinary teaching programme. When introducing students to the field of energy conversion and system dynamics, no systematic approach concerning the use of analogies has been attempted so far. However, it is possible to offer a basis for a generalized introduction into energy conversion using energy as a universal quantity which links the whole spectrum of energy forms. Any specific field built around a particular energy can then be used as a starting point leading into adjoining fields by the use of analogous relations, hence a systematic approach offers a considerable simplification in the teaching procedures, and the potential understanding of learning processes and knowledge transfer.

THE PAPER concentrates on a specific educational approach used in the introduction of fundamental concepts of energy conversion demonstrating the use of analog principles to ease information transfer processes in interdisciplinary fields. This approach relies mostly on an interesting phenomenon known in psychology as transfer of knowledge. It has been found in educational research that previously acquired knowledge influences the process of introducing new information, specifically where analogous models are used. In this particular case, the application of simple physical models is extremely useful in the formation of basic concepts and principles of energy conversion.

To highlight the theoretical considerations concerning the method applied for this purpose, some simple examples of models, in which transfer of knowledge is particularly visible are demonstrated and thoroughly examined, and the implications on the teaching process in engineering education are discussed. The paper discusses not only the mechanisms of transfer appearance and the analogous quantities which are common in the models shown but what is even more important, it also endeavours to categorize those features of the analogous models used which are of particular importance in the teaching-learning of basic concepts of engineering and science.

INTRODUCTION

Energy distribution uniquely defines the state of a physical system. Any change in the state of a system is then associated with a change in energy distribution, and as a consequence, results in energy conversion processes. Since there are many energy forms involved, we are particularly concerned with a host of direct and indirect energy couplings and conversions in which a unique and exclusive system of analogy emerges.

The relations describing various conversion processes are very similar and thus lead to many analogous relationships. Such analogies have been discussed widely in literature [1–8]. By using energy as a basic function linking all energy forms, it is possible then to establish a systematic approach leading to analogue relations which are also philosophically associated.

It is feasible to set up a variety of alternative analogue connections relating to specific phenomena. However, the energy-based approach is unique and leads to a generalized system of energy-based relations, handling the whole spectrum of energy forms at a macroscopic level. This inevitably leads to an interdisciplinary approach to teaching of energy conversion in all branches of engineering and science, and thus making the transfer of knowledge more manageable.

ANALOGUE RELATIONSHIPS

Analogue relations for general physical systems can be derived directly from the universal nature of energy. These relations lead to what may be defined as a generalized theory which provides a general basis to which all analogies can be referred. There are a number of fundamental and unique relations for energy. Energy in a system is conserved and thus one form of energy is related directly to another form of energy. This, and the fact that energy in a closed system always seeks a

minimum, allows us to establish system performance relations. These take the same mathematical form for all energies, hence they are analogous.

The analogous nature of system equations has been extensively discussed in literature in the past, relating to specific situations [1–4]. Electrical and mechanical analogies are the most common relations and they have been used extensively in control system applications and analogue system simulation for many years now. For instance, analogue computers make extensive use of these features in dynamic systems.

As another example, thermal systems are often simulated using electrical analogies. It is possible to use different analogous sets of variables suitable for particular situations. A natural analogous set is presented in Table 1. This particular natural set is shown to relate the temperature T to the voltage e, and the entropy S to the electrical charge q. The analogy shown is energy based, and hence is called natural, since it can be extended to other energy forms.

Table 1. Terms—Electrical analogue sets

Natural	Special
$T \sim e$	$T \sim e$
$S \sim q$	$Q \sim i$
$P \sim f$	
$V \sim x$	$Q = kT; i = ge$
	k = thermal conductivity
	g = electrical conductivity

A special alternative analogue set relating T to e, and the thermal energy flux Q to the electric current i, is particularly easy to use for studies involving thermal heat conductivity. It is very useful to demonstrate heat flux relations in terms of electric current flows. However, this analogous set is restricted to this particular set of variables and therefore lacks flexibility, when other energy forms are involved.

To establish the energy based relation, we have to consider the basic laws of energy conversion, and hence to deduce the generalized performance relationships.

BASIC CONCEPTS

A basic set of postulates defining the fundamental energy function and a set of energy states can be established covering the full spectrum of known energy forms when dealing with macroscopic phenomena. This paper offers an attempt to extend this postulatory approach and shows that it provides a basis ranging from thermodynamics to general systems theory. For energy studies, it provides a unified approach which should make the teaching process simple.

Established statistical mechanics and thermody-namics have provided a framework from which theories for specific areas such as solid state physics and the theory of liquids, can be deduced. Specific theories involve simplifications and restrictions that have to be imposed on the postulatory framework offered.

Energy conversion statics goes beyond thermodynamics and provides a postulatory formulation which employs states of a system rather than processes. Energy and entropy, as well as derived state functions, feature as a starting point and lead to statements about Carnot cycles, irreversibility, etc. It demonstrates the universal character of the statements as, for example, the fact that the Carnot cycle is a property [1–4] transcending classical thermodynamics and applying to any pair of coupled energy forms not necessarily involving thermal energy.

A fundamental approach to the theory of energy, energy conversion and transfer, as developed here, must predispose the maroscopic consequences of statistical mechanics. This allows the setting up of lumped parameter characterization of phenomena and leads to conventional and more special theories like electro-mechanics.

Attempts to establish comprehensive methods for general solution of problems concerned with energy systems have been described by Mela *et al*. [13–14]. In ref. [13], Rayleigh's dissipation function is used as a starting point. This approach derives from the entropy maximum criterion as discussed in [4]. In Ref. [14], energy relations are introduced and used to set up analogue relations on a systematic basis.

In this paper, we set out the basic relations as already discussed in literature [1–3] and extend this to allow for non-static situations usually dealt with under the umbrella of irreversible thermodynamics. It is shown how the basic state functions and state laws for coupled energy sources are derived leading to the generalized system relations in terms of partial derivatives of the system energy function. As follows, for example, the thermodynamic energy function leads to fundamental relations and concepts like the Carnot Cycle, which apply to any other coupled energy pair. Using the postulatory approach, a host of theorems can be derived [3] setting out the performance criteria for energy systems.

ENERGY FUNCTION

The energy stored in a system depends on the work done on, and extracted from, the system. This storage energy is described by an energy function U which is defined by four postulates as already discussed [1–4].

We are concerned here with macroscopic systems which relate generally to agglomerates of large numbers of nuclear components. Energy conservation applies to the components hence we can assume that a definite energy can be assigned to the macroscopic systems. The resulting energy

function of a system is then characterized by postulating [1]

1. the equilibrium state as a function of extensive variables,
2. the minimum energy criterion,
3. the mathematical restrictions imposed on the energy function and
4. the requirement that the energy of a system must approach zero when the intensive variables or forces are zero.

Extensive and intensive variables are defined later.

A system may be made up of a number of subsystems. The total energy is then the sum of that of all subsystems. Energy distribution defines the state of a system. Any change in this distribution involves a change of state and this is accompanied by energy conversion or transfer processes.

In general, we are dealing with two situations, firstly a steady state or quasi-static condition as defined by the postulates and, secondly, a situation involving non-equilibrium conditions which are usually dominated by loss processes in which energy is degraded into a state of lower availability.

Energy is made up of available and unavailable energy defined as exergy and unergy. Given the energy of a closed system U we have then:

$$U = E + B$$

where

E = exergy which is available for conversion into other energy forms

B = unergy which is unavailable for conversion.

A closed system is defined as an isolated system with no energy exchange across its boundaries and total energy is conserved but not exergy.

Although energy is universally conserved, exergy continuously decreases as energy is degraded. At the same time, unergy continuously increases and this corresponds in many ways with the increase in entropy of the environment postulated by the second law of classical thermodynamics, however, it is a more definite statement. The terms exergy and unergy have been introduced in literature to help to distinguish first law and second law efficiency [1] and we have, in general:

$$\frac{\partial E}{\partial t} < o; \frac{\partial B}{\partial t} > o$$

Instead of using entropy as a starting point, we are here using energy and the concepts of exergy as the basis for the setting up of a general theory for energy systems. This approach leads to the theory of mechanics, electro-mechanics and modern system theory dealing with macroscopic phenomena. These areas involve Hamilton's principle and the energy minimum criterion as basic concepts. In following this approach, we can introduce thermodynamics as a special case. In contrast, classical thermodynamics uses the entropy maximum principle with entropy as the fundamental state function. Since entropy is usually ignored in electro-mechanics and system theory, an entropy based approach leads to difficulties.

One has to distinguish here between concentrated and distributed systems. In the concentrated representation we consider energy entering a system at specified points of entry or ports. This leads to port representation and lumped parameter description which is nearly universally adopted in mechanics and system theory. Distributed systems are concerned with volume flows which have to be analysed in multi-dimensional forms. They can be described in terms of simple parameters but only in the limit when dealing with small variations in system quantities.

In this paper, we are dealing with lumped parameter representation of elements for energy storage and conversion and for the coupling of energy sources and sinks.

EXERGY AND UNERGY VERSUS ENTROPY

The major areas of energy conversion and transfer such as electromechanics, which are covering vast fields of technology, have been developed and can be readily treated without the use of the concept of entropy. However, we know that entropy is generated in any process involving heat losses and degradation of heat.

Losses are generally allowed for in conservative system theory simply by introducing loss elements [1–9]. This is not so simple in thermodynamics, since losses in thermodynamics appear as a somewhat diffuse heat loss or as heat degradation and this is difficult to separate in practice from reversible heat fluxes [10].

Energy flows and flux quantities are usually of distributed nature. However, they are very often assumed to be concentrated into ports of entry in multi-entry storage elements. We can then resort to lumped parameter representations for the coefficients describing couplings or linkage between ports and storage elements.

When dealing with loss processes the distributed nature becomes more pronounced and simple lumped parameter representation becomes more difficult. Loss processes are usually studied assuming small changes for a first approximation. They develop usually when quasi static equilibrium is disturbed by changing forces.

For example, when voltage across a resistive element rises, an electric current starts to flow. This current is trying to re-establish the original equilibrium by reducing the voltage. At the same time, the current creates entropy.

Similarly, when a temperature gradient is applied across a thermo-electric element, a heat flow develops and this is associated by an entropy creation or flux and is trying to bring temperatures down. At the same time, an electron drift produces a voltage which results in the thermo-electric effect.

Thus, the change in forces introduces a new reversible coupling effect which is caused by the forces and not by their rate of change as in the case of the usual and quasi-static coupling effects [1]. This important difference will emerge when setting up the general interrelations as discussed in more detail later.

MULTIPORT THEORY

Energy forms

Energy is a concept or construct and any specific energy form is defined by two physical quantities. Any such two quantities are defined as an associated pair made up of a force and a flux or, in mathematical terminology, as an intensive and an extensive quantity as shown in Table 2. There are, for example, four directly coupled sets of such associated pairs dealing with macroscopic phenomena:

- potential and kinetic energies
- chemical and caloric energies
- electric and magnetic energies
- nuclear and radiation energies.

There can be indirect coupling between pairs.

Pairs of associated quantities are shown in Table 2 and the related energy increments are given. In the case of coupling of several energy sources, we apply the requirement of energy conservation. Four typical coupling situations are shown in Table 3. They all represent *two-port systems* where two energy sources are coupled via a joint storage element (Fig. 1).

Basic energy relations

In any system, energy must be conserved and its behaviour is governed by two general laws. If changes occur, the energy seeks a minimum, and this is another physical requirement. In particular, the useful energy decreases with time, if the system is left on its own. If work is done, or energy is introduced into a system, the storage energy U must increase and if work increments are given as dW_i the change in U must be dU or

$$dU = \Sigma dW_i. \tag{1}$$

However, some losses occur during such a process, and in practice we have

$$dU \leqslant \Sigma dW_i. \tag{2}$$

Here the relation (1) is the law of energy conservation, whereas the relation (2) is the law of energy dissipation.

In Table 2, the associated pairs for the general range of energies are shown [2]. In Table 3, the

Table 3. Energy storage for typical pairs of couples sources

Magneto-Mechanical System $dU = id\psi + fdx$	ψ = magnetic flux linkage i = electric current f = mechanical force x = displacement
Caloric System $dU = TdS - PdV$	T = temperature S = entropy P = pressure V = volume
Electro-Caloric System $dU = Tds + edq$	e = voltage q = charge
Magneto-Caloric System $dU = TdS + Hdm$	H = magnetic intensity M = magnetization

$$dW_1 = y_1 dx_1 \qquad dW_2 = y_2 dx_2$$
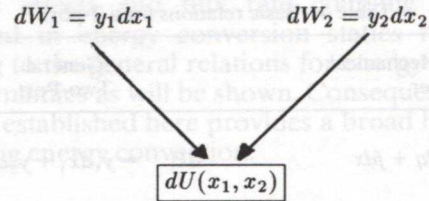
$$\boxed{dU(x_1, x_2)}$$

$$dU(x_1, x_2) = y_1 dx_1 + y_2 dx_2$$

Fig. 1. Two-port diagram.

Table 2. Sets of associate variables

System	Force or Intensive Variable	Flux or Extensive Variable	Energy Increment	Power
Mechanical				
potential	f force	x displac.	fdx	fx
kinetic	$\dot{x}$ speed	p momentum	$\dot{x}dp$	$\dot{x}\dot{p}$
Thermal				
caloric	T temp.	S entropy	TdS	$T\dot{S}$
chemical	g chem. potent.	N number density	gdN	$g\dot{N}$
mechanical	P press.	V volume	$-PdV$	$-P(dV/dt)$
Electro-magnetic				
electric	e volt.	q charge	edq	$e\dot{q}$
magnetic	i curr.	ψ flux linkage	$id\psi$	$i\dot{\psi}$
Nuclear				
radiation	E ener. level	ϕ photon density	$Ed\phi$	$E\dot{\phi}$
nuclear	μ nuclear force	N number density	μdN	$\mu\dot{N}$

coupling relations, as determined by the first law, are shown [3] for four typical examples. In Table 4, the performance relations for an electro-mechanical transducer are derived and generalized.

The similarity of these relations is then made evident by introducing the concept of port-representation. In such cases, we are dealing with two coupled energy sources and have then, what is defined as a two-port system, as depicted in Fig. 1.

A very relevant example is shown in Table 5, where the conventional relationships of two magnetically coupled electrical coils are derived from the energy functions. The energy function represents the energy stored in the field produced

Table 4. Basic relations for a 2-port

Electro-Mechanical Transducer	General Two-Port
Law I	
$dU = edq + fdx$	$dU = y_1dx_1 + y_2dx_2$
Law II	
$dU \leqslant edq + fdx$	$dU \leqslant y_1dx_1 + y_2dx_2$
Equations of State	
$U = U(q,x)$	$U = U(x_1, x_2)$
$e = \dfrac{\partial U(q,x)}{\partial_q}$	$y_1 = \dfrac{\partial U(x_1, x_2)}{x_1}$
$= e(q,x)$	$= y^1(x_1, x_2)$
$f = \dfrac{\partial U(q,x)}{\partial_x}$	$y_2 = \dfrac{\partial U(x_1, x_2)}{x_2}$
$= f(q,x)$	$= y_2(x_1, x_2)$
Performance Equations	
$de = U_{qq}dq + U_{qx}dx$	$dy_1 = U_{11}dx_1 + U_{12}dx_2$
$df = U_{xq}dq + U_{xx}dx$	$dy_2 = U_{21}dx_1 + U_{22}dx_2$
$U_{qx} = U_{xq}$	$U_{x_1x_2} = U_{x_2x_1}$

Table 5. Transformer relations

Law I $dU = i_1d\psi_1 + i_2d\psi_2$	
Law II $dU \leqslant i_1d\psi_1 + i_2d\psi_2$	

Equations of State

$$U = U(\psi_1, \psi_2) \quad i_1 = \frac{\partial U(\psi_1, \psi_2)}{\partial_{x1}} \quad i_2 = \frac{\partial U(\psi_1, \psi_2)}{\partial_{x2}}$$

Performance Equations

$$di_1 = U_{11}d\psi_1 + U_{12}d\psi_2$$
$$di_2 = U_{21}d\psi_1 + Y_{22}d\psi_2 \quad ; U_{12} = U_{21}$$

or inverting

$$d\psi_1 = F_{11}di_1 + F_{12}di_2 = L_{11}di_1 + M_{12}di_2$$
$$d\psi_2 = F_{21}di_1 + F_{22}di_2 = M_{21}di_1 + L_{22}di_2$$

These relations prove that mutual inductance must be equal since

$$F_{21} = F_{12} \rightarrow M_{12} = M_{21} = M$$

Since coefficients are constants these equations can be integrated, i.e. they are *holonomic*,

$$\psi_1 = L_{11}i_1 + Mi_2$$
$$\psi_2 = Mi_1 + L_{22}i_2$$

by the currents i_1 and i_2 in the coils. The flux linkage of each coil is given by ψ_1, and ψ_2. This particular example is interesting, since it demonstrates the simplest way in which to prove that the mutual inductances for the two coils, as seen by either, are identical, i.e. $M_{12} = M_{21}$.

Derived energy functions

As shown in Table 5, for the transformer relations we had to introduce a new energy function $F(i_2, i_2)$ to establish an inverted relation. This leads to the concept of derived energy functions, as widely used in Classical Thermodynamics and Dynamics.

For example, the derived and so-called free energy functions of thermodynamics, i.e. the enthalpy H, free energy F and Gibbs energy G are a special class of a more general class of derived energy functions (see Table 6(i)). This general form has been defined as the general class of Helmholtz functions for n-ports: $F(y_i; x_j)$, where $i = 1, \ldots, r$, and $j = 1, \ldots r + 1, \ldots n$.

Table 6. Derived energy functions and potential relations

(i) Thermo-Dynamics	
dU	$= TdS - PdV$
dH	$= TdS + VdP$
dF	$= -SdT - PdV$
dG	$= -SdT + VdP$
(ii) Electro-Mechanics	
dU	$= id\psi + fdx$
dF_H	$= id\psi - xdf$
dF_F	$= -\psi di + fdx$
dF_G	$= -\psi di + xdf$
(iii) Thermo-Electrics	
dU	$= TdS + edq$
dF_H	$= TdS - qde$
dF_F	$= -SdT + edq$
dF_G	$= -SdT - qde$

Here n is the number of sources and ports of the system described by the Helmholtz function. The Helmholtz functions are based on the energy function $U(x_i; x_j)$ which is equivalent to the Hamiltonian of a system. The Helmholtz functions are derived using a simple Legendre transformation such that:

$$dF(y_1, \ldots, y_k, x_{k+1} \ldots, x_n) = dU(x_1, \ldots, x_n) - d(x_1y_1, \ldots, x_ky_k). \tag{3}$$

This class of function includes the Lagrangian, Co-Lagrangian and widely used co-energy functions of dynamics and electro-mechanics.

In thermodynamics, we are generally concerned with a single energy storage medium, i.e. a two-port system. For two coupled sources there exist three

derived Helmholtz functions only leading to the well-known free energies (see Table 6(i)).

In dynamics we generally deal with systems of *n*-ports, hence we have multiple storage elements and systems leading to a multiplicity of possible derived energy functions. A more systematic approach to the selection of classes of Helmholtz functions is then necessary, such as the Lagrangian and Co-Lagrangian types, and their variations used throughout electro-mechanics [1–6]. In some ways, the Co-energy type function used in electrical machine theory corresponds to the Gibbs free energy of the thermodynamic two-port. The problem of systems involving mixed variables has been discussed in the past by Kron [11] and others.

ENERGY CONVERSION STATICS AND CONSERVATIVE SYSTEMS

The basic derived energy function used in mechanics and dynamics is the Lagrangian state function which is a Legendre transformed energy function derived from the Hamiltonian. The Hamiltonian is the total energy of a system and as stated before corresponds with the energy function of energy conversion statics.

It is important to note that the Lagrangian is not a function of the extensive variables only, but a mixed function. This fact was not important in earlier work in mechanics which dealt with linear mechanical systems. In electro-mechanics the following relations exist then for the Hamiltonian H and Lagrangian L:

$$H = U \tag{4}$$
$$= K + V$$
$$L = -F_L(y_l, \ldots, y_r, x_{r+1}, \ldots x_n) \tag{5}$$
$$= K' - V$$

where

$$V = V(x_{r+1}, \ldots, x_n)$$
$$\quad = \text{Potential energy function}$$
$$K = K(x_1, \ldots, x_r)$$
$$\quad = \text{Kinetic energy function}$$
$$K' = K'(y_1, \ldots, y_r)$$
$$\quad = -[K(x_1, \ldots, x_r) - d(x_1 y_1 + \ldots + x_r y_r)]$$
$$\quad = \text{Co} - \text{kinetic energy function}$$
$$F_L = \text{derived Helmholtz function}$$
$$\quad = -\text{Lagrangian L.}$$

In electro-mechanics K' is the co-kinetic and V the potential energy of a system. The roles of K' and V can be exchanged when changing from loop to nodal relations or vice versa in a systems study [4].

The Langrangian state function becomes useful in interconnected systems in electromechanics. Introducing mixed variables via the ko-kinetic energy function, it is possible to simplify the system relations and reduce the number of equations [1]. The negative sign in L allows for the fact that a Legendre transformation changes the sign for the resulting equations of state because of the change in

the dependent variables. When applying Kirchhoff's nodal or mesh laws the Lagrangian emerges naturally [1].

In the case of a coupled thermo-chemical system of energy forms we are commonly dealing with the Gibbs energy function which can be directly related to the Lagrangian function. This fact indicates the close relationship between electro-mechanics and thermostatics.

The problems of allowing for loss effects have never been satisfactorily resolved in conservative dynamics. Rayleigh's loss function must make use of an artificial factor of two [3, 5]. In addition flux rate cross couplings have never been allowed for as discussed below.

Loss effects, and flux rate coupling can be included in energy conversion statics relations leading to the general relations for energy conversion dynamics as will be shown. Consequently, the theory established here provides a broad basis for handling energy conversion.

GENERAL SYSTEM THEORY

Reversible conditions

For a general *n*-port the energy increment is given by

$$dU(x_1, x_2 \ldots x_n) = y_1 dx_1 + y_2 dx_2 + \ldots y_n dx_n. \tag{6}$$

Introducing matrix notation

$$dU(x_1, \ldots, x_n) = Y_t dX = \sum_{i=1}^{n} y_i dx_i$$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_2 \end{bmatrix} \text{ and } Y_t \text{ is transpose of } Y$$

$$dX = \begin{bmatrix} dx_1 \\ \vdots \\ dx_2 \end{bmatrix}$$

and Y_t is the transpose.

The state laws are then given by the partial derivatives

$$\frac{\partial U(x_1, \ldots, x_n)}{\partial x_i} = U_{x_i} = U_i = Y_i$$

and the force increments or differential performance relations are

$$dy_1 = U_{11}dx_1 + U_{12}dx_2 + \ldots + U_{1n}dx_n \tag{7}$$

$$dy_n = U_{n1}dx_1 \ldots + U_{nn}dx_n.$$

where the U_{ij} elements are the second order derivatives of U, i.e.

$$U_{m1} = \frac{\partial^2 U(x_1, \ldots, x_n)}{\partial x_m \partial x_1} = \frac{\partial^2 U(x_1, \ldots, x_n)}{\partial x_1 \partial x_m}$$

Thus in matrix form we have

$$dY = U_{ij}dX$$

where $U_{ij} = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ U_{21} & U_{22} & \cdots & \\ \cdots & & \cdots & \\ U_{n1} & & \cdots & U_{nn} \end{bmatrix}$

The inversion relation leads to

$$dX = U_{ij}^{-1}dY \tag{8}$$

where

$$D_{ij} = U_{ij}^{-1}.$$

We also have the Legrende transformation

$$\begin{aligned} F(y_1, \ldots, y_n) &= U(x_1, \ldots, x_n) - (x_1 y_1 + \ldots \\ &\quad + x_n y_n) \\ &= -D(y_1, \ldots, y_n) \end{aligned}$$

i.e. the inverse energy component matrix can be expressed as

$$D = -F \quad \text{or} \quad D_{ij} = -F_{ij}.$$

Thus D_{ij} is closely related to co-energy functions. So far only static changes in the associated quantities have been considered. In practice flux rates become important when considering energy flow or power. For example, the charge q is the extensive quantity of generalized flux for a capacitor. The flux rate would be the current i or

$$i = \frac{dq}{dt}$$

In general the flux rate matrix is

$$I = \frac{d}{dt} X = \dot{X}.$$

Thus the reversible flux rate relations follow from relation (8) above

$$\begin{aligned} I_{rev} &= D_{ij} \frac{d}{dt} Y \\ &= D_{ij} \dot{Y}. \end{aligned} \tag{9}$$

Rate processes

When forces are applied to a system, flux rates appear and a non-equilibrium state usually develops. This leads, for example, to thermal conduction and electrical resistance drops. It can also involve flux rate coupling effects like thermo-electricity, piezoelectricity, etc. We have for these effects the Onsager relations which express the fact that these phenomena depend on forces directly and not the rate of change of forces as shown above for reversible conditions. Thus the flux rates are given by;

$$I_{rate} = R_{ij} Y \tag{10}$$

where $R_{ij} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & & & \vdots \\ \vdots & & & \\ \alpha_{n1} & & \cdots & \alpha_{nn} \end{bmatrix}$

and R_{ij} is the rate coefficient matrix.

We get the total flux rates including reversible and rate processes using (2) and (3):

$$\begin{aligned} I &= I_{rev} + I_{rate} \tag{11} \\ &= D_{ij} \frac{d}{dt} Y + R_{ij} Y \\ &= \left(D_{ij} \frac{d}{dt} + R_{ij} \right) Y \\ &= G_{ij} Y \end{aligned}$$

and here $G_{ij} = D_{ij}(d/dt) + R_{ij}$ is analogous to an impedance matrix.

For steady state the flux rate relation reduces to:

$$I = R_{ij} Y \tag{12}$$

which is equivalent to the Onsager relations [10].

Power flow

The power flow into the system is given by the sum of the power flows from the ports. This is:

$$\begin{aligned} P &= Y_t I \\ &= Y_t G_{ij} Y \\ &= Y_t D_{ij} \dot{Y} + Y_t R_{ij} Y. \tag{13} \end{aligned}$$

If we are dealing with a linear system, the first term takes a more familiar form by taking d/dt outside

$$\begin{aligned} P &= \frac{1}{2} \frac{d}{dt} (Y_t D_{ijo} Y) + Y_t R_{ij} Y \tag{14} \\ &= \frac{d}{dt} D_o + R \end{aligned}$$

where

$$\begin{aligned} D_o &= \frac{1}{2} Y_t D_{ijo} Y \\ &= \text{a derived energy function} \\ R &= Y_t R_{ij} Y \tag{15} \\ &= \text{rate function}. \end{aligned}$$

The rate function includes loss terms which cause a production in unergy.

The power relations here is sufficient when analysing purely electrical and mechanical systems and systems without entropy sources. It is evident that conventional dynamic system relations can only be established when the H_{ij} coefficients are time invariant.

There is still one component missing when considering the power relation in general systems and that is the heat flux due to heat generation by electrical loading and resistive heating in electrical systems, friction and other irreversible processes producing heat and causing entropy or unergy to rise. This can be allowed for by an entropy production source term:

$$P_{entr} = S_{ij} Y.$$

The total power is then:

$$P = Y_t D_{ij} \dot{Y} + Y_t R_{ij} Y + Y_t S_{ij} Y. \tag{16}$$

where the last term is a rate of production of unergy due to losses in addition to those included in R.

State space relations

The performance relations can now be re-arranged:

$$D_{ij} \dot{Y} = -R_{ij} Y + S_{ij} Y + I$$

This relation can be put into the form:

$$\dot{Y} = AY + BI \tag{17}$$

where

$$A = -D_{ij}^{-1}(R_{ij} + S_{ij})$$
$$B = D_{ij}.$$

This is then the state space form of the performance relations. It is a simple matter to establish these relations in particular cases using D_{ij} and R_{ij} and making allowance for the entropy production term.

Interconnection

When simple components are interconnected, we impose constraints on variables and reduce the number of independent quantities. This is covered in detail in circuit theory where topogical relationships are set out specifying interconnection requirements for meshes and modes. The same applies for mechanical and other lumped parameter systems.

In a particular situation, usually a number of sets of independent variables can be chosen as specified by the topological relationship between branches, nodes and meshes. If a set I is chosen and we want to change to a new set I_u then this is related by a connection matrix C and the Ys by M:

$$I_u = CI \tag{18}$$
$$Y_u = MY.$$

The consequential relations between the forces can be derived using the concept of power invariance. This implies that fluxes in individual elements in such a system do not change due to a different choice of variables. In that case we have:

$$Y_{ut} I_u = Y_t I$$

or

$$Y_{ut} CI = Y_t I$$

thus

$$Y_{ut} C = Y_t$$

or

$$C_t Y_u = Y$$
$$= M^{-1} Y_u$$

Hence

$$M = C_t^{-1}$$

and

$$Y_u = C_t^{-1} Y \text{ if } |C| \neq 0. \tag{19}$$

This relation is well known for orthogonal transformations when invariant constraints are imposed. The transformation can be extended to cases when further interconnection reduces the degrees of freedom in the system. The connection matrix then becomes rectangular and has no inverse, but the force connection M can still be related to the flux connection matrix C.

CONCLUSION

Energy conversion theory can be based on the energy function and the energy minimum principle making use of the concepts of exergy and unergy. This differs from the conventional entropy based approach used in classical thermodynamics.

The energy function can then be defined within the framework of four basic postulates [3] and system equations can be established leading to systematic relationships as used in dynamics and electro-mechanics.

It is important to note the restrictions, generally unstated, imposed on derived energy function. For example, the conventional dynamics, the Lagrangian as used, applies only for linear systems. In general, one must use the Co-Langrangian state function. Another feature is that the theory of classical thermodynamics reduces to a special case of two-port or more generally n-port theory.

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