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Using CAD Software to Analyze Spatial Mechanisms*

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This paper discusses three-dimensional analysis of mechanisms by using the CAD (Computer Aided Drafting) software. The method is shown to be effective and highly accurate. An RSSR *(Revolute-Sphere-Sphere-Revolute) example is given.*

INTRODUCTION

IT HAS been shown in literature that implementing the graphical method on the CAD system is an effective way to perform two-dimensional mechanism analysis [1,2,3,4]. This paper is dedicated to three dimensional analysis of mechanisms on the CAD system.

There are some advantages of using the graphical method on the CAD system over the analytical method. First, the graphical method, being very accurate on the CAD system, requires a lower level of mathematics for the user to understand and to apply. Second, with the graphical method, we can easily visualize, on the computer monitor, the mechanism and the motion vectors from different projection angles in a solution process. Third, while phase-wise the analytical method can give us the motion history in one run of a program, the graphical method can be more straightforward point-wise in the way that it is very convenient to consider the motion characteristics of different points on a mechanism with the pointing device of the CAD system and by applying velocity-image principle and the acceleration-image principle.

Different from two-dimensional analysis, the graphical method for three-dimensional mechanism would not be practicable without use of the CAD system. The amount of effort to implement the three-dimensional graphical method manually can be prohibitively large. With the CAD system, we can verify the magnitudes, the directions, and the inter-relations of the geometric elements very conveniently, and hence, results are reliable.

The CAD systems in our lab are Computervision microCADDS GCD 3.0. The work can also be performed on other types of CAD systems that have three-dimensional capability.

This paper addresses the effectiveness of the implementation through an *RSSR* mechanism, which has analytical solutions available in a well adopted textbook on spatial mechanisms.

ANALYSIS ON THE CAD

With the graphical method, the underlying problem in rigid body kinematic analysis is to solve vector equations such as

$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \ldots + \vec{V}_m = \vec{V}_{m+1} + \vec{V}_{m+2} + \vec{V}_{m+3} + \ldots + \vec{V}_n$$

where the vectors may represent positions, velocities, or accelerations. In general all the vectors are not in the same plane.

The 3-D CAD system uses a concept called the construction plane (CPL). A CPL is a suitably defined local *xoy* plane, which enables us to construct geometric elements in terms of local *x-y-z* coordinates. Use of the CPL is very essential in three-dimensional construction. For example, let us draw a line through a point perpendicular to a plane in space. Using the CPL, we can select the given plane as the construction plane and draw a line in the local *z* direction through the given point. A CPL has its associated view plane which presents the full view of the CPL.

Local coordinates, however, are not stored in the run-time memory of a computer running CAD software for retrieval in a later time of an implementation.

ANALYSIS OF AN *RSSR* MECHANISM

For comparison purpose, we illustrate the graphical approach to an *RSSR* mechanism discussed in a textbook by C. H. Suh *et al*. with the analytical method [5, p. 79].

Kinematic drawing of a general *RSSR* mechanism is shown in Fig. 1. Figure 2 shows the simplified kinematic drawing of the particular *RSSR* mechanism, where link *AB* is the crank, link *BC* is

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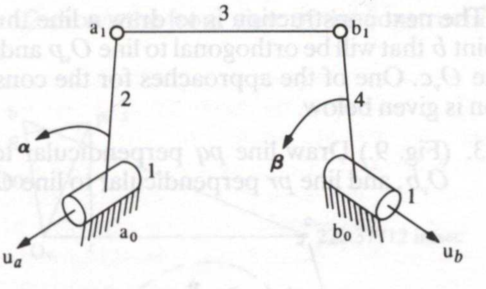

Fig. 1. RSSR mechanism.

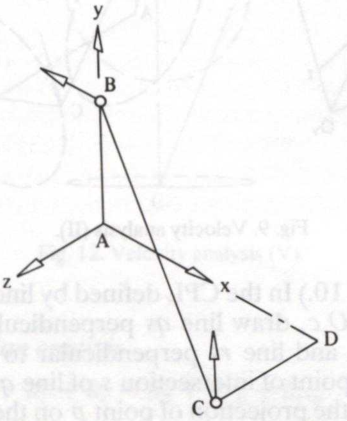

Fig. 2. Kinematic drawing of an RSSR mechanism.

the coupler, and link CD is the follower. A global Cartesian coordinate system has been defined and the coordinates of the joints in the mechanism are $A = (0.,0.,0.)$ in., $D = (2.,0.,0.)$ in., $B = (0.,1.,0.)$ in. and $C = (2.,0.,1.)$ in. at the initial time. The axes of the revolute joints at A and D are on the z-axis and the x-axis, respectively. Joints B and C are spherical pairs. The problem is to find the position, the velocity and the acceleration of point C when the crank has rotated 10°, assuming that it has an angular velocity of 100. rad/sec, and an angular acceleration of −10. rad/sec^2 at the angle.

Displacement analysis

The construction steps follow.

1. (Fig. 3.) Rotate the crank from its initial position AB_o to the 10°position AB.

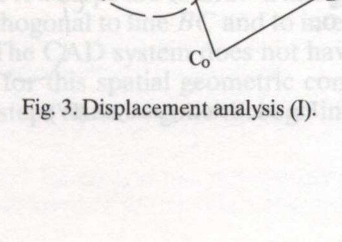

Fig. 3. Displacement analysis (I).

2. (Fig. 4.) Construct the projection of point B on the plane of motion of the follower. For this mechaism, we could draw line BO parallel to line AD and draw line DO parallel to line AB_o. The two lines meet at point O, which is the projection of point B.

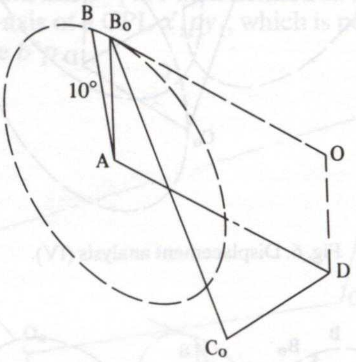

Fig. 4. Displacement analysis (II).

Construction of the projection of a point on a plane in a general case will be shown in the next section.

3. (Fig. 5.) In the motion plane of the follower, construct a circle centered at point O with its radius

$$R = \sqrt{B_oC_o^2 - BO^2}.$$

Each point on the circle has a distance to point B equal to the length of the coupler B_oC_o.

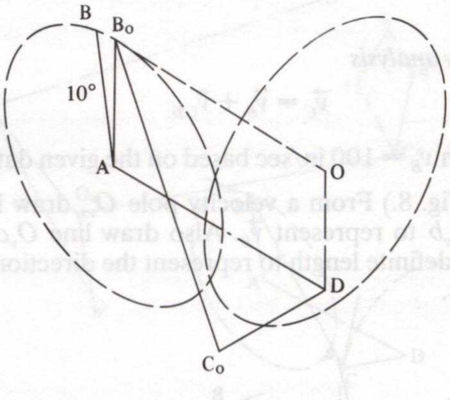

Fig. 5. Displacement analysis (III).

4. (Fig. 6.) In the same plane, construct another circle centered at point D and passing through point C_o. This reflects the constant length condition of the follower.

The point of intersection of the two circles defines point C, the new location of the end point of the follower.

The angle between line DC_o and line DC is measured to be 20.6497° (Fig. 7). The solution in [5, p. 81] is 20.65°.

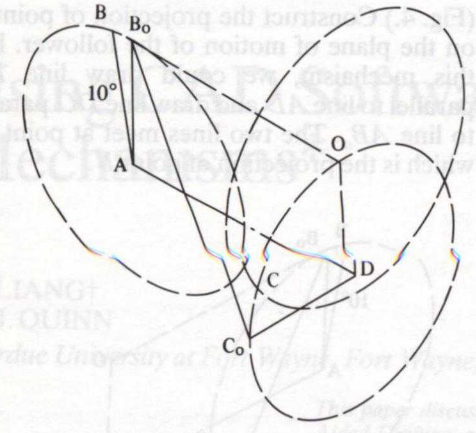

Fig. 6. Displacement analysis (IV).

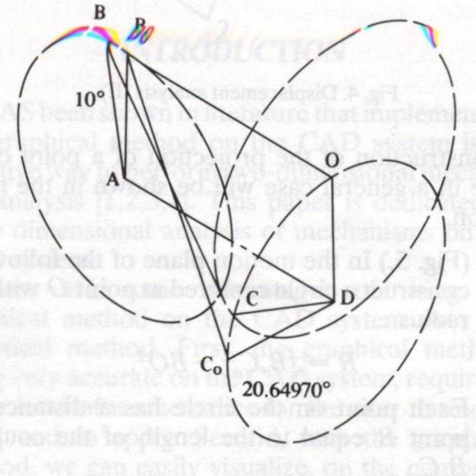

Fig. 7. Displacement analysis (V).

Velocity analysis

$$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$$

in which $v_B = 100$ in/sec based on the given data.

1. (Fig. 8.) From a velocity pole O_v, draw line O_vb to represent $\vec{v}_B$. Also draw line O_vc of indefinite length to represent the direction of v_C.

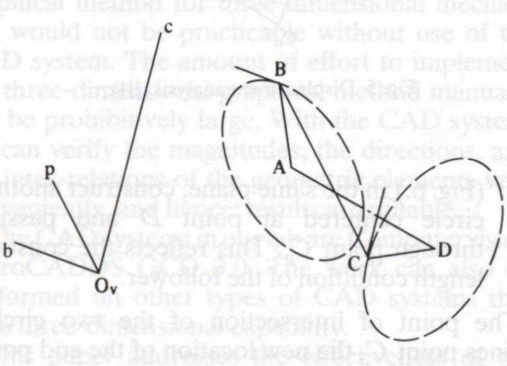

Fig. 8. Velocity analysis (I).

2. Draw line O_vp of arbitrary length parallel to line BC.

The next construction is to draw a line through point b that will be orthogonal to line O_vp and meet line O_vc. One of the approaches for the construction is given below.

3. (Fig. 9.) Draw line pq perpendicular to line O_vb, and line pr perpendicular to line O_vc.

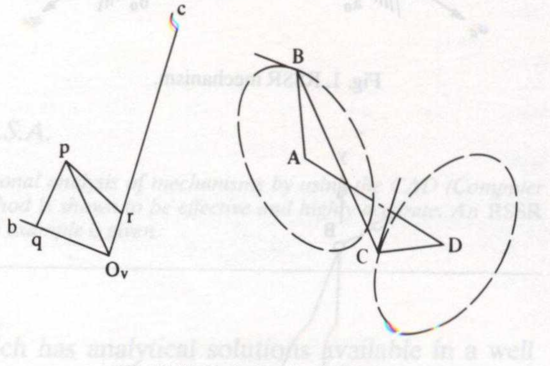

Fig. 9. Velocity analysis (II).

4. (Fig. 10.) In the CPL defined by line O_vb and line O_vc, draw line qs perpendicular to line O_vb, and line rs perpendicular to line O_vc. The point of intersection s of line qs and line rs is the projection of point p on the CPL.

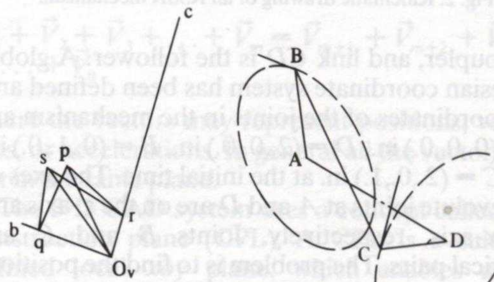

Fig. 10. Velocity analysis (III).

5. (Fig. 11.) In the same CPL, draw line bc perpendicular to line O_vs. Line bc meets line O_vc at point c. Note that line bc is orthogonal to line BC.

Velocity at point C represented by line O_vc is found to be 220.37712 in/sec. The answer in [5, p. 81] is .220E+03 in/sec.

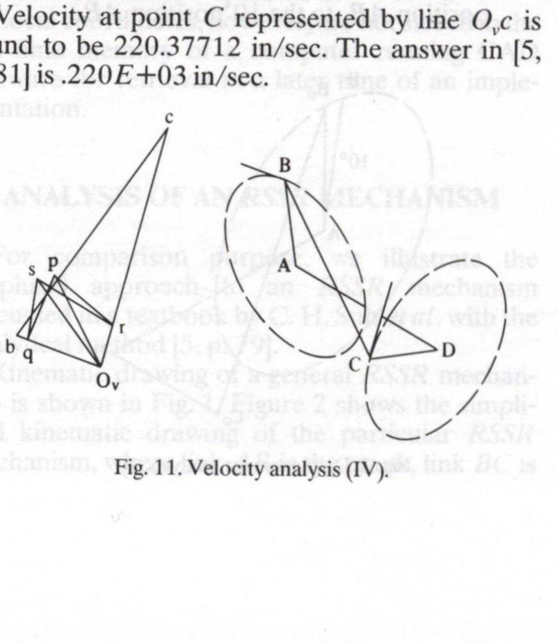

Fig. 11. Velocity analysis (IV).

Fig. 12 shows the front view of the construction plane.

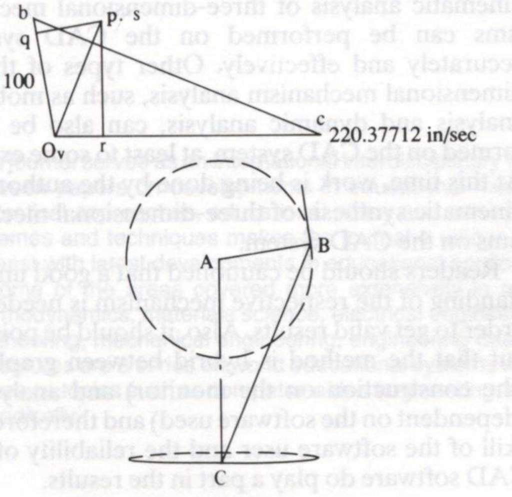

Fig. 12. Velocity analysis (V).

Acceleration analysis

$$\vec{a}_C^n + \vec{a}_C^t = \vec{a}_B^n + \vec{a}_{C/B}^n + \vec{a}_{C/B}^t$$

where, based on the given data and the results of the velocity analysis, $a_C^n = 220.37712^2$ in/sec^2, $a_B^n = 100^2$ in/sec^2, $a_B^n = 10$ in/sec^2, and $a_{C/B}^n = 256.37410^2/2.44949$ in/sec^2. It can be seen that, compared with a_B^n, a_B^t is small enough to be neglected.

Steps of the construction are as follows.

1. (Fig. 13.) From an acceleration pole O_a, draw line $O_a b'$ for $\vec{a}_B$ which is essentially equal to $\vec{a}_B^n$ in this problem, and draw line $O_a c^*$ for $\vec{a}_C^n$.
2. Draw line $b'p$ to represent $\vec{a}_{C/B}^n$, and draw line c^*q of indefinite length in the direction of $\vec{a}_C^t$.

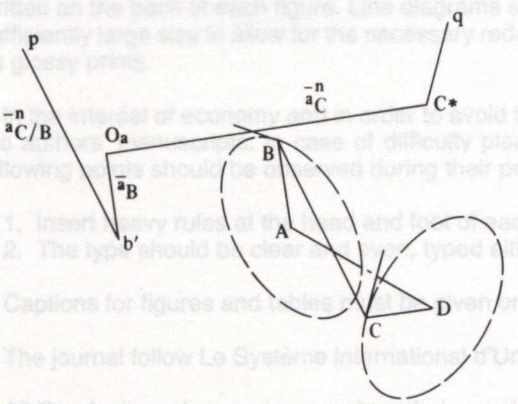

Fig. 13. Acceleration analysis (I).

The next few steps are to draw a line from point p to be orthogonal to line BC and to intersect with line c^*q. The CAD system does not have a single command for this spatial geometric construction, and a few steps must be gone through instead. We choose to first construct, through point p, a plane perpendicular to line BC, and then locate the point of intersection of the plane and line c^*q.

3. (Fig. 14.) Draw line py_1 perpendicular to the plane of line $b'p$ and line $O_a p$, and draw line px_1 perpendicular to the plane of line $b'p$ and line py_1. Line px_1 and line py_1 are then defined as the x-axis and the y-axis of a CPL $x_1 p y_1$, which is perpendicular to line $b'p$.

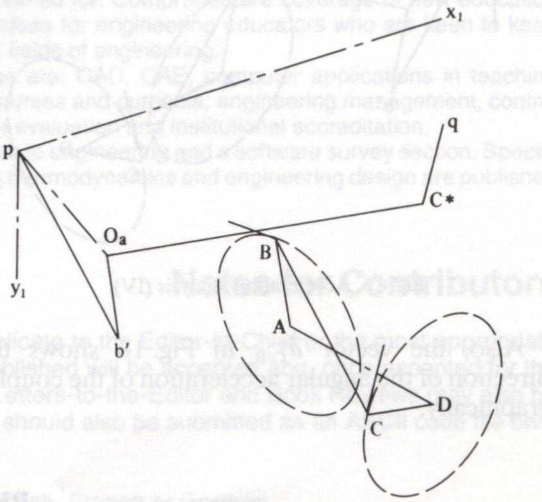

Fig. 14. Acceleration analysis (II).

4. (Fig. 15.) Construct the points of projection r and s of points c^* and q on the CPL.

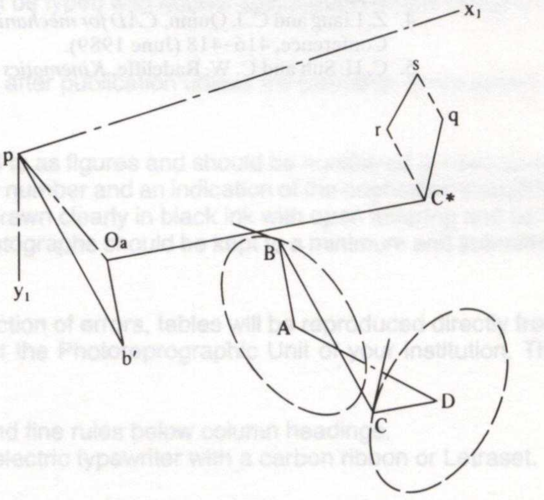

Fig. 15. Acceleration analysis (III).

Some details of the construction for the project of a point on a plane were shown in the previous section, Velocity analysis.

5. (Fig. 16.) The extensions of line rs and line c^*q meet at point c'. c^*c' is the vector $\vec{a}_C^t$.

The angular acceleration of the follower is found to be 26052.220 rad/sec^2. The solution [5, p. 81] is $261E+0.5$ rad/sec^2.

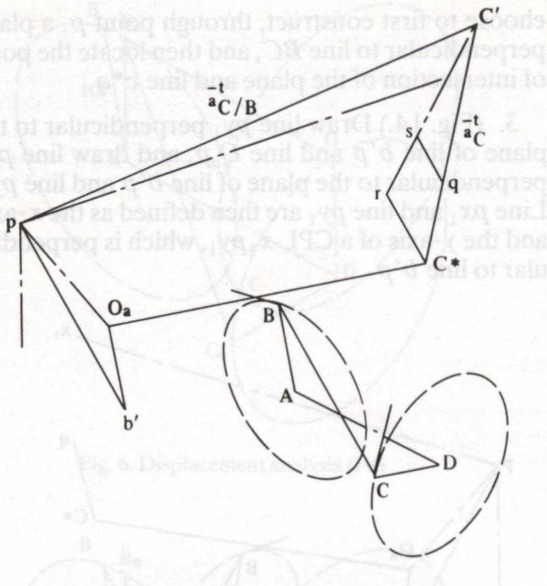

Fig. 16. Acceleration analysis (IV).

Also, the vector $\vec{a}^{\,t}_{C/B}$ in Fig. 16 shows the direction of the angular acceleration of the coupler graphically.

CONCLUSION

The paper shows in a very convincing way that kinematic analysis of three-dimensional mechanisms can be performed on the CAD system accurately and effectively. Other types of three-dimensional mechanism analysis, such as mobility analysis and dynamic analysis, can also be performed on the CAD system, at least to some extent. At this time, work is being done by the authors on kinematic synthesis of three-dimensional mechanisms on the CAD system.

Readers should be cautioned that a good understanding of the respective mechanism is needed in order to get valid results. Also, it should be pointed out that the method is hybrid between graphical (the construction on the monitor) and analytical (dependent on the software used) and therefore the skill of the software user and the reliability of the CAD software do play a part in the results.

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